Accepted for publication in The Astrophysical Journal

Ages for illustrative field stars using gyrochronology: viability, limitations and errors

Sydney A. Barnes

Lowell Observatory, 1400 W. Mars Hill Rd., Flagstaff, AZ 86001, USA

barnes@lowell.edu

ABSTRACT

We here develop an improved way of using a rotating star as a clock, set it using the Sun, and demonstrate that it keeps time well. This technique, called gyrochronology, permits the derivation of ages for solar- and late-type main sequence stars using only their rotation periods and colors. The technique is clarified and developed here, and used to derive ages for illustrative groups of nearby, late-type field stars with measured rotation periods. We first demonstrate the reality of the interface sequence, the unifying feature of the rotational observations of cluster and field stars that makes the technique possible, and extends it beyond the proposal of Skumanich by specifying the mass dependence of rotation for these stars. We delineate which stars it cannot currently be used on. We then calibrate the age dependence using the Sun. The errors are propagated to understand their dependence on color and period. Representative age errors associated with the technique are estimated at ~15% (plus possible systematic errors) for late F, G, K, & early M stars. Ages derived via gyrochronology for the Mt. Wilson stars are shown to be in good agreement with chromospheric ages for all but the bluest stars, and probably superior. Gyro ages are then calculated for each of the active main sequence field stars studied by Strassmeier and collaborators where other ages are not available. These are shown to be mostly younger than 1 Gyr, with a median age of 365 Myr. The sample of single, late-type main sequence field stars assembled by Pizzolato and collaborators is then assessed, and shown to have gyro ages ranging from under 100 Myr to several Gyr, and a median age of 1.2 Gyr. Finally, we demonstrate that, in contrast to the other techniques, the individual components of the three wide binaries ξBooAB, 61CygAB, & αCenAB yield substantially the same gyro ages.

Subject headings: open clusters and associations: general — stars: activity — stars: evolution — stars: late-type — stars: magnetic fields — stars: rotation

1. Introduction

1.1. Stellar age indicators, and motivation for a rotation clock

The age of a star is its most fundamental attribute apart from its mass, and usually provides the chronometer that permits the study of the time evolution of astronomical phenomena. Consequently, a great deal of effort has been expended over the past several decades on the possibility of using stars as clocks, to reveal their own ages, those of the astronomical bodies associated with them, and to understand how various astronomical phenomena unfold over time.

The most successful of these chronometric techniques is the isochrone method (invented by Sandage, 1962; named and developed substantially by Demarque and Larson, 1964), based on the steady change in the color-magnitude morphology of a collection of stars, in response to the consumption and dimunition of their nuclear fuel (e.g. VandenBerg et al., 2006; Demarque et al., 2004; Girardi et al., 2002; Kim et al., 2002; Yi et al., 2001).

1.1.1. The principal limitations of the Isochrone clock

The isochrone technique fashions a collection of coeval stars of differing masses, i.e. a star cluster, into a remarkable clock that provides the age of the system. However, vast numbers of stars, including our own Sun and most of the nearby stars amenable to detailed study, are no longer in identifiable clusters and spend their lives in relative isolation as field stars. For these stars, the isochrone technique is less useful, because a star spends most of its life burning hydrogen steadily on the main sequence, where its luminosity and temperature, the primary indicators of isochrone age, are almost *constant*[1]. Using classical isochrones to tell the ages of single, low-mass, main sequence stars is akin to using gray hairs or baldness as an age indicator for toddlers, adolescents and adults!

Furthermore, the isochrone technique requires a measurement of the distance to a field star to calculate its luminosity. This distance is hard to measure, and in fact, even after the publication of the results of the Hipparcos satellite (ESA, 1997), we know the distances to only $\sim$20,000 field stars (all of them nearby) to better than 10% (Perryman et al. 1997). This imprecision leads to large errors in isochrone ages. A 10% error on the distance to a solar twin would result in $\sim$20% errors in its luminosity, and isochrone ages between 2 and 10 Gyr[2]. Because the age of a star provides a direct link to many of its other properties, this deficiency is keenly felt. Knowledge of the age of a field star, however crude, is a very valuable astronomical commodity indeed.

Thus we need to consider the possibility of fashioning clocks using other properties of (individual) stars. In particular, it would be very valuable to construct an age indicator that is independent of distance, and indeed, some of the activity-related indicators suggested over the years, including the primary one used today, do have this valuable characteristic. In fact, the details of the pros and cons of the isochrone and other chronometers are such that it might be useful here to step back even further and consider how an age indicator is constructed, and the general characteristics desirable for stellar age indicators.

1.1.2. Steps in the construction of age indicators

Five major steps seem to describe the process:

1. One needs to find an observable, v, that changes sensitively and smoothly, perhaps monotonically, with age. Preferably, this observable would be a property of individual stars rather than that of a (co-eval)

[1]For example, in the 4.5 Gyr since it was on the zero age main sequence (ZAMS), the Sun's luminosity has increased by $\leq$50% of its ZAMS value.

[2]The Hipparcos satellite has indeed provided $\sim$1% parallaxes for a group of stars, most of them bright enough to have been included in the catalog of Hipparchus himself if they were visible from Greece! To count them, you would need your own digits and those of some of your collaborators, but you wouldn't need more than a few of the latter.

collection of them.

2. One needs to determine the ages of suitable calibrating objects independently. These would provide the connection to the fundamental units like earth rotations, pendulum swings, etc.

3. One needs to identify and measure the functional form of the variable: $v = v(t, w, x, ...)$ where t is the age, and w, x, ..., are possible additional dependencies of the variable v. It is preferable to have fewer variables, and to have separable dependencies of the form $v = T(t) \times W(w) \times X(x)$.....

4. One needs to invert the dependence determined experimentally, numerically or otherwise, to find $t = t(v, w, x, ...)$. This is usually non-linear, and sometimes has undesirable kinks.

5. Finally, one needs to calculate the error $\delta t = \delta t(t, v, w, x, ...)$.

1.1.3. Characteristics desired for age indicators

The foregoing considerations suggest that the following characteristics are desirable for stellar age indicators.

1. **Measurability for single stars:** The indicator should be properly defined, measurable easily itself, and preferably should not require many additional quantities to be measured, otherwise it cannot be used routinely.

2. **Sensitivity to Age:** The indicator should have a sensitive dependence on age, i.e., should change substantially (and preferably regularly) with age, otherwise the errors will be inherently large.

3. **Insensitivity to other parameters:** The indicator should have insensitive (or separable) dependencies on other parameters that affect the measured quantity, otherwise there is the potential for ambiguity.

4. **Calibration:** The technique should be calibrable using an object (or set of objects) whose age(s) we know very well, otherwise systematic errors will be introduced.

5. **Invertibility:** The functional dependence determined above should be properly invertible to yield the age as a function of the measured variables.

6. **Error analysis:** The errors on the age derived using the technique ought to be calculable, otherwise no confidence can be attached to the ages.

7. **Test of coeval stars:** The technique should yield the same ages for stars expected to be coeval, otherwise the validity of the technique itself must be questioned.

We summarize in Table 1 how (in)adequately these characteristics are satisfied by the three age indicators relevant to this paper. While the entries, especially for gyrochronology, anticipate the results derived in this paper, the characteristics desired guide the progress of, and form a continuous backdrop to this work.

1.1.4. Motivations for investigating a rotational clock

A wide array of age indicators have been developed over the past decades. The most well-known are chromospheric emission (Wilson, 1963) and rotation (Skumanich, 1972), but others like surface lithium abundance (Vauclair, 1972; Rebolo, Martin & Maguzzu 1992; Basri, Marcy & Graham 1996; Stauffer 2000) and coronal emission in X-rays (Kunte et al. 1988), usually through its dependence on rotation (Pallavicini et al. 1981; Gudel 2004), have also occasionally been suggested and used in various contexts. All of these are related to stellar activity and are based on empirical correlations between the property in question and stellar age. They have been considered less reliable clocks than the canonical isochrone technique because the underlying physics is not well understood, and in fact there is a great deal of debate even about what the important underlying phenomena are. Finally, one must also consider whether and how each of these age indicators is calibrated.

Ever since the work of Skumanich (1972), and especially since the work of Noyes et al. (1984), the relationship between chromospheric emission and age has enjoyed the distinction of being the most consistent, making chromospheric emission the leading age indicator for nearby field stars (e.g. Soderblom 1985; Henry et al. 1996; Wright et al. 2004).

But there are more fundamental stellar observables than chromospheric emission. In fact, of all the activity-related properties of stars, rotation is undoubtedly the most fundamental, and many believe that together with stellar mass (and another variable or two), it might be responsible, directly or indirectly, for the observed morphology of all the other activity indicators.

In fact, besides being obviously independent of the distance to the star, stellar rotation is now known to change systematically, even predictably, on the main sequence, where the isochrone technique is at its weakest. Furthermore, the specific form of the rotational spindown of stars is such that *initial variations in the rotation rates of young stars appear to become increasingly unimportant with the passage of time, leading to an almost unique relationship between rotation period and age for a star of a given mass.* Finally, rotation is a property we can now measure to great precision; rotation periods for late-type stars are sometimes determined today to better than one part in ten thousand[3]! These features of stellar rotation - its predictability, measurability, and simplicity - suggest that some effort is warranted in improving its use as an age indicator beyond the relationship suggested by Skumanich (1972).

In fact, as we shall show below, and as is summarized in Table 1, gyrochronology satisfies more of the criteria required for an age indicator as listed above, than any other astronomical clock in use, and appears to be complementary to the isochrone technique, in that it works very well on the main sequence, while the isochrone method is better suited to evolved stars.

This paper addresses the issues of constructing and calibrating a rotational clock. It appears that to first order stellar rotation depends only on the mass and age of the star, so that jointly taking account of these dependencies of rotation permits the determination of rotational ages (*and their errors*) for a substantial sample of main sequence stars, and even *individual field stars*, a technique we suggest be called "gyrochronology."

---

[3]The usefulness of this precision is less clear in the context of the differential rotation with latitude of the Sun and solar-type stars, but it is also clear that we are beginning to understand the systematics and origin of differential rotation, so that the attainment of such precision is useful in other ways as well.

Table 1. Characteristics of the three major age indicators for field stars

Property	Isochrone age	Chromospheric age	Gyrochronology
Measurable easily?	?[a] (Distance reqd.)	?[b] (Repetition reqd.)	?[c] (Repetition reqd.)
Sensitive to age?	No (on MS)	Yes	Yes
Insensitive to other parameters?	No	Yes[d]	Yes
Technique calibrable?	Yes (Sun)	?[e] (Sun?)	Yes (Sun)
Invertible easily?	No	Yes	Yes
Errors calculable/provided?	?[f] (Difficult)	Yes?[g]	Yes
Coeval stars yield the same age?	No (Field binaries)	?[h]	Yes

[a]A field star requires a good distance measurement in order to determine its luminosity for comparison with isochrones. As explained in the text, good distances are available to only a few such stars.

[b]Another reason for this '?' is that it is not clear to an innocent bystander how to transform between the various quantities published as a chromospheric flux: S, HK index, R, or R'_{HK}. The lack of a defined standard quantity for published work is a significant drawback.

[c]Another reason for this '?' is that for old stars the modulation in broadband photometric filters is too small to yield a rotation period, and for these stars one must resort to more onerous means such as detecting the rotational modulation in the Ca II H & K emission cores.

[d]The benefit of doubt has been given but in fact, there is usually some black magic in the transformation between chromospheric flux and age.

[e]The relationship between chromospheric emission and age in Soderblom et al. (1991) is calibrated against isochrone ages of three "fundamental" points, and those of the evolved components of visual binaries. Since all isochrones are calibrated using the age of the Sun, this calibration is also ultimately based on the age of the Sun, except for the additional step involved.

[f]Errors on isochrone ages for field stars were essentially non-existent until Pont & Eyer (2004) suggested a Bayesian scheme that allows one to determine whether or not an isochrone age is 'well-defined' (Jorgensen & Lindegren 2005) i.e. whether or not the probability density distribution for the age has an identifiable maximum, and if so, to calculate an error based on this property. This method has since been used by Takeda et al. (2007) on their field star sample.

[g]Soderblom et al. (1991) provide the error on their fit, in this case $\sim$0.17 dex ($\sim$40%), of chromospheric emission to (isochrone) age for their sample. Other researchers, including Donahue (1998), usually do not provide errors.

[h]For the eight pairs in Table 2 of Donahue (1998), the mean discrepancy is 0.85 Gyr for a sample with a mean age of 1.85 Gyr, so that the fractional discrepancy in age is 0.46, or just under 50%.

1.2. Stellar rotation as an astronomical clock

Major steps in the direction of using stellar rotation as a clock were made by a series of studies in the 1960s, culminating in the famous relationship of Skumanich (1972), relating the averaged surface rotational velocities, $\overline{v \sin i}$, of stars in a number of open clusters to their ages, t, via the expression: $\overline{v \sin i} \propto 1/\sqrt{t}$. Skumanich noted that the equatorial surface rotation velocity of the Sun at its independently derived age also matched this relationship[4]. Over the years, astronomers have come to believe that this relationship encapsulates something fundamental about the nature of winds and angular momentum loss from late-type stars[5].

Skumanich (1972), however, did not specify the mass-dependence of rotation - the so-called 'correction for color' that he performed. Presumably this correction was based on the Kraft (1967) curve or something similar. There is also a measurement issue - for individual stars the ambiguity inherent in using $v \sin i$ measurements, with the generally unknown angles of inclination, i, can be expected to introduce large errors in the age determinations[6].

Furthermore, mass-dependent comparisons of rotation require precise values for stellar radii to infer the true angular rotation speeds of stars. Despite these shortcomings, various studies have occasionally used this relationship for rotational ages, e.g. Lachaume et al (1999), and the ages derived in this manner are in rough agreement with ages derived using other techniques, but they are not noticeably better.

Kawaler (1989) attempted an empirical color correction using a linear function of the $(B - V)$ color of the star, but he provided no physical basis for such a correction - indeed, there is none - and in any case it breaks down dramatically for late-F to early-G stars (see especially Fig. 1 in his paper). The specific ways in which stars of different masses spin down, whether young clusters obey such a spindown or not, and how observations in young clusters are related to field star observations is a continuing matter of debate and discussion.

If it were possible to eliminate the ambiguity in $v \sin i$ observations by finding the true angular rotation rates of stars, as is routinely accomplished nowadays by measuring rotation periods[7], and if the periods were to have a unique and "correctable" dependence on color, with reasonably small scatter, rotation could become incredibly useful as a stellar clock.

Using the (measured) rotation periods of the Mt. Wilson stars, Barnes (2001) showed that the age

---

[4]The age of the Sun is not directly known, of course. We use the age of the formation of the refractory inclusions in the Allende meteorite as an estimate of the Sun's age (e.g. Allegre et al. 1995 but see also Patterson 1953; 1955; 1956; Patterson et al. 1995 and Murthy & Patterson, 1962 for the original work establishing that the age of the Earth and that of the meteorites is identical and can be called the "age of the solar system").

[5]It appears to be equivalent to a cubic dependence on the rotation speed, Ω, of the angular momentum loss rate, dJ/dt, from solar- and late-type stars: $dJ/dt \propto -\Omega^3$ (Kawaler, 1988). In fact, parameterizations based on this behavior are routinely incorporated into stellar models that include rotation (e.g. Pinsonneault et al 1989). Two of these three powers of Ω appear to be related to the strength of the magnetic field of the star under the assumption of a linear dynamo.

[6]Projection effects are less relevant for entire clusters, as with the averaged $v \sin i$ measurements that Skumanich used. Presumably they average out because they are similar from cluster to cluster.

[7]Rotation periods are measured by timing the modulation of either filtered starlight, which works well for young stars (e.g. Van Leeuwen et al. 1987), or that of the chromospheric emission (e.g. Noyes et al. 1984), which works for older stars. Either of these is obviously more demanding than deriving $v \sin i$, but the effort is well worth the results, and furthermore, is being done routinely, as detailed below. As an aside, we point out that the "rotation periods" listed by Wright et al. (2004) are *not* directly measured; they are calculated from the measured chromospheric emission, and hence unsuitable for our purposes.

dependence of rotation for these stars is indeed Skumanich-type ($P \propto \sqrt{t}$), and furthermore, the mass dependence of rotation for these stars is similar to that observed in the Hyades open cluster. Barnes (2003a) noted that an age-increasing fraction of open cluster stars and essentially all solar-type stars beyond a few hundred Myr in age, including individual field stars, obeyed the same mass dependence. These two facts provide the connection between rotation in clusters and in the field.

Furthermore, Barnes (2003a) wrote down this mass dependence, f, as a convenient function[8] of $(B-V)$ color, $f(B-V)$. This function, f, appears to be closely related to the moment of inertia, I_*, of the entire star via $f \propto 1/\sqrt{I_*}$. This identification, the rotational implications for the Sun and cluster stars, and for stellar magnetic fields, are discussed at length in Barnes (2003a and 2003b), but here we are concerned only with the *universality and uniqueness* of this function, apparently separable from the age dependence, a circumstance that leads to a remarkably simple way of deriving ages (and their errors) for solar-type stars on the main sequence[9].

1.3. Proximate motivations for constructing a rotational clock

There are also proximate motivations for this work. It has become increasingly obvious that greater precision in stellar ages than is available using isochrones and chromospheric emission is required for many astronomical purposes. The effort currently being expended on the host stars of planetary systems is a case in point. Well determined ages would eventually permit the study of the the evolution of planetary systems. This application is a proximate one relevant to our time, but the method can undoubtedly be used to tackle some of the deeper problems in astronomy.

The requirement of a stellar rotation period is not as onerous as might initially appear[10]. *As opposed to the requirement for isochrones, it avoids the necessity of deriving the distance to a field star.* The Vanderbilt/Tennessee State robotic photometric telescopes (e.g. Henry et al. 1995) and of the University of Vienna (e.g. Strassmeier et al. 2000) in Southern Arizona are designed to derive stellar rotation periods, and in fact, the Strassmeier group, now in Potsdam, has almost finished the construction of two 1.2m telescopes, Stella 1 & 2, to monitor active stars almost exclusively (Strassmeier, 2006). The ASAS project (e.g. Pojmanski, 2001) routinely monitors and catalogs stellar (and other) variability in the Southern hemisphere, and a Northern counterpart is the Northern Sky Variability Survey (Wozniak et al. 2004).

The Canadian MOST satellite (Matthews et al. 2000) was launched to provide (and has since delivered) superb time-series photometry (witness its identification of two closely spaced rotation periods for κ^1 Ceti, corresponding to two spot groups; (Rucinski et al. 2004), its detection of 0.03%-0.06% brightness variations in a subdwarf B star; (Randall et al. 2005), and its recent identification of g-modes in β CMi (Saio et al. 2007)). The COROT satellite mission has been designed[11] to study stellar convection, rotation, and now, planetary transits. A number of ground-based telescopes are planning to or already exploiting the time

---

[8]He used the the function: $f(B-V) = \sqrt{B-V-0.5} - 0.15(B-V-0.5)$ but f can of course be written in terms of any convenient function of stellar mass. We will modify the expression for f below.

[9]I have learned from Ed Guinan (2006, personal communication) that he has been using the Hyades rotational sequence and the Skumanich relation to derive stellar ages. That would make it substantially similar to the technique developed here.

[10]We note here that chromospheric emission measurements also require repeated measurement to ensure that they are averages over the variability from rotation or from stellar cycles.

[11]In fact, the satellite has been built and launched.

domain, and of these the Large Synoptic Survey Telescope (LSST) perhaps has the greatest visibility.

The Kepler space mission, being readied for launch, is likely to yield not only the planetary transit, but also the rotation period of the host star. In fact, Kepler is likely to yield rotation periods for orders of magnitude more stars than planetary transits[12]. Regardless of whether or not the Kepler mission delivers what it promises, stellar rotation periods will be determined routinely as time domain astronomy comes into its own. A very significant portion of time-domain work on stars will yield the stellar rotation period (it is a by-product of all searches for planetary transits), and if this measurement can be used to derive a precise stellar age, it would permit us to address many problems involving chronometry that are not presently solvable.

1.4. Overview of the paper and sequence of succeeding sections

Our goal here is to specify the stars for which gyrochronology can and cannot be used, to develop it to yield useful ages for individual field solar-type stars, and to calculate the errors on these ages. We will also show that where both are available, these new ages agree with (and might even supercede) the ages provided by other methods.

We begin by showing that rotating stars, whether in clusters or in the field, are of two types[13]: fast/Convective/C and slow/Interface/I. The Sun is shown to be on the interface sequence, which defines the rotational connection between all solar-type stars (section 2). These stars are shown to spin down Skumanich-style, with a mass dependence that is shown to be universal, and for which we derive a simple functional form using stars in open clusters (section 3). These functional dependencies are combined to yield a simple expression for the gyro ages of stars.

In section 4, we derive the errors on these ages. Section 5 demonstrates that these ages compare favorably with chromospheric ages for a well-studied sample. Sections 6 and 7 illustrate the use of gyrochronology on samples for which other ages are not uniformly available, namely the field star samples of Strassmeier et al. (2000) and Pizzolato et al. (2003). Section 8 demonstrates that gyrochronology yields the same age for the two component stars of wide binaries. Section 9 contains a comparison to recently derived isochrone ages for a common subset of the stars considered here, and Section 10 contains the conclusions.

2. The rotational connection between all solar-type stars

A fundamental fact of stellar rotation is that there are two major varieties, C & I, of rotating solar-type (FGKM) stars (see Barnes 2003a). A third variety, g, merely represents stars making an apparently unidirectional transition from one variety (C) to the other (I). All three varieties of stars are normally found in young open clusters, but the Sun and all old solar-type stars are of only the I variety. Each of these varieties of rotating stars has separate mass- and age dependencies that can be clarified considerably merely by effecting the correct separation of the stars by variety. One of these, called the Interface (I) sequence stars, containing the Sun and all old field solar-type stars, is related to the property Skumanich noticed in 1972. This group is the one that we will use here to demonstrate the technique of gyrochronology, because

---

[12] Assuming that we do not throw the baby out with the bath water.

[13] In principle, there is a third type, g, representing stars in transition from the first/C to the second/I type.

stars change into this variety over time. We are fortunate that the rotational mass- and age dependencies of this group of stars appear to be both separable and also particularly simple.

If the mass- and age dependencies of this sequence are indeed separable, as was claimed by Barnes (2003a) to be of the form $P(t, M) = g(t).f(M)$, then merely dividing the measured rotation periods $P(t, M)$ by the functional form $g(t)$ of the age dependence should make the mass dependence $f(M)$ manifestly clear. For observational convenience, and also to avoid the error inherent in the conversion from $B - V$ to stellar mass, we have used $f(B - V)$ instead of $f(M)$. Removing an assumed Skumanich-type age dependence, where $g(t) = \sqrt{t}$, is particularly simple, and appears to bring the I sequence into sharp focus, leading to the identification of the mass dependence as a function $f = f(B - V)$. In the two subsections below, we effect this determination separately for cluster and field stars.

2.1. The connection between clusters themselves

Here we show that f represents the connection between most rotating stars, and that the functional dependence of f on color or stellar mass is common to all open clusters. We use all the open cluster rotation periods currently available in the literature; note that we are restricted to those stars for which $(B - V)$ colors are also available. The major sources are listed in Table 2. We divide each of the measured rotation periods by $g(t) = \sqrt{t}$ where t is the age of the cluster in Myr, as listed in Table 2. These quantities are plotted against de-reddened $(B - V)$ color in Fig. 1 , on a linear scale in the upper panel, and on a logarithmic scale in the lower panel. Clusters are color-coded violet through red in increasing age sequence.

The most striking aspect of these data is the curvilinear feature representing a concentration of stars in the vicinity of the solid line. This is the interface sequence, I, proposed in Barnes (2003a), the one that will consume our attention in this paper, and whose position we will use as an age indicator for field stars. Along the bottom of the upper panel one may also discern another linear concentration of stars which represents the convective sequences, C, of the youngest open clusters. This sequence could also potentially be used as an age indicator for young stars, but its dependencies on stellar age and mass are more complicated than those of the I sequence (see Barnes 2003a), and we do not use it here. Stars located between these sequences are either on the convective sequences of the older open clusters in this sample, or in the rotational gap, g, between the interface and convective sequences.

Every single cluster plotted in Fig. 1 possesses an identifiable interface sequence. The fraction of stars on this sequence increases systematically with cluster age, as shown earlier in Barnes (2003a); see especially Fig. 3 there. But for us the crucial feature of these data is that these age-corrected sequences overlie one another. This feature is shared by all open clusters, and can be represented by a function $f(B-V)$, common to all clusters. A particular choice (used in Barnes 2003a) of $f(B-V) : \sqrt{(B-V-0.5)} - 0.15(B-V-0.5)$ is displayed in both panels. This is of the nature of a trial function, useful in locating the I sequence roughly, and we will improve on this choice subsequently.

Barnes (2003a) has suggested that f ought to be identified with $1/\sqrt{I_*}$, where I_* is the moment of inertia of the star, implying a substantial mechanical coupling of the entire star on this sequence. The suggestion in that publication was magnetic coupling by an interface dynamo, hence the name interface sequence for this group of stars.

The dotted lines in the figure are drawn at $2f$, and $4f$. Present indications are that some of these stars are either non-members of the cluster, sometimes stars with spurious/alias periods or otherwise misidentified variables of another sort. Note that the Hyades, where excellent membership information is available, has no stars above the sequence. A similar situation obtains in NGC 2516 and M 34, which are also relatively clean samples. Good cluster membership information could resolve this issue completely.

In summary, the behavior of the open cluster rotation observations suggests the existence of a feature common to all open clusters, the Interface sequence, which is observationally definable by its common mass dependence, $f(M)$, across clusters, here represented by $f(B-V)$. These observations also justify the use of the Skumanich (1972) relationship between rotation and age to describe the age dependence of rotation, but *only for rotating stars of this particular (interface) type.*

Table 2. Principal sources for open cluster rotation periods.

Cluster	Age	Rotation Period Source
IC 2391	30 Myr	Patten & Simon (1996)
IC 2602	30 Myr	Barnes et al. (1999)
IC 4665	50 Myr	Allain et al. (1996)
Alpha Per	50 Myr	Prosser & Grankin (1997)
Pleiades	100 Myr	Van Leeuwen, Alphenaar & Meys (1987), Krishnamurthi et al. (1998)
NGC 2516	150 Myr	Barnes & Sofia (1998)
M 34	200 Myr	Barnes (2003a)
NGC 3532	300 Myr	Barnes (1998)
Hyades	600 Myr	Radick et al. (1987)
Coma	600 Myr	Radick, Skiff & Lockwood (1990)

2.2. The connection between clusters and field stars

Here we show that the mass dependence, f, among open clusters, is also shared by field stars as exemplified by the Mt. Wilson stars. We begin by removing from the Mt. Wilson sample those stars known or suspected not to be dwarfs (based on Baliunas et al. 1995), to avoid any possible complexity related to structural evolution off the main sequence. An effective connection with open clusters requires splitting the remaining main sequence Mt. Wilson field star sample by age, to control the age variation among the stars and gain leverage over the time domain. Fortunately, one such split, based on detailed studies of this sample, especially of chromospheric emission, has already been made by Vaughan (1980), who classified these stars into a Young (Y) and Old (O) group. The simplest course of action is to use the existing divisions. Although the age divisions are quite broad, subsequent work has confirmed the basic classification. A cut by chromospheric activity is well-known to be also a cut by rotation and age (e.g. Barnes 2001, and references therein). Part of the goal of this paper is to develop a way of ordering the stars by age, so we cannot start by assuming chromospheric ages for individual stars.

As a result of the above classification, we have two groups of stars, Y and O, consisting of 43 and 49 stars respectively, equivalent to two additional open clusters, each containing stars with a wide range of ages. What are these ages? Barnes (2001) [see especially Fig. 3] and Barnes (2003a) [see especially Fig. 2] suggest that, in terms of rotation, the young (Y) stars range in age from less than 300 Myr to about 2 Gyr, with a characteristic age of 800 Myr, while the old (O) stars range in age from 2 Gyr to about 10 Gyr, with a characteristic age of 4.5 Gyr. The age of the Y group is older than, but comparable to, young open cluster ages, while the age of the O group is reasonably represented by the Sun's age. Effectively, we are assuming that the Sun is an appropriate representative, in rotation and age, of the Old Mt. Wilson sample.

If we use the chromospheric ages for the same Y and O groups of Mt. Wilson stars, calculated using the relationship of Donahue (1998), the median ages of the same samples work out to be 780 Myr and 4.24 Gyr respectively, reasonably close to our assumption above. We will use these new values as the representative ages for the Y and O groups in this paper. The rotation clock can easily be recalibrated when the need arises.

We can make the field star data comparable with the open cluster data by similarly removing this approximate age dependence. Thus, we divide the rotation periods of the Young Mt. Wilson stars by $\sqrt{t_Y = 780\,Myr}$ and display them using small black asterisks in Fig. 2, overplotted on the open cluster data (colored circles). Similarly, we divide the rotation periods of the Old Mt. Wilson stars by $\sqrt{t_O = 4240\,Myr}$ and display them in Fig. 3 using large black asterisks, again overplotted on the open cluster data.

Examination of Figs. 2 and 3 leads to several conclusions. Firstly, we note that both the Young and Old samples overlie the Interface sequences of the open clusters. The greater dispersion of the Y and O stars relative to those of the open cluster I sequences can be traced to the age dispersion in each of these samples. We can see, despite this dispersion, that C sequence stars, and possibly g (gap) stars, are absent from both the Young and Old samples. These data are consistent with all of the Mt. Wilson stars being of the I variety.

Any doubts about the classification of the Young and Old Mt. Wilson stars can be settled by making individual corrections for these stars based on their chromospheric ages. If these ages are correct, then removing their dependence, as in the open clusters, should make the mass dependence obvious, and that mass dependence ought to be similar to f. In fig. 4 we display the result of dividing the Mt. Wilson star rotation periods by the square root of the (individual) chromospheric ages, calculated using the formula from

Donahue (1998)[14]. We note that almost all of the Mt. Wilson stars in the color range considered lie on/near $f(B-V)$. Fig. 4 displays $f(B-V)$ (solid line), $0.8f$ and $1.25f$ (dotted lines), to show this proximity. Indeed, a free-hand fit would be almost identical to f. We will improve on this trial function below. It is likely that this sample does not contain any C sequence stars or even any gap stars. (This observation is consistent with the C→I transition timescale of ∼200 Myr observed in open clusters.)

In closing this section, we reiterate that the rotation period distributions of open clusters, when corrected for a Skumanich-type age dependence, display two strong concentrations of stars. The slower of these consists of sequences that are common to all open clusters and overlie one another. Rotation period distributions of main sequence field stars, despite the difficulty of correcting for their ages, also display this same sequence. The other concentration of stars, present in open clusters, is absent here. The feature common to cluster and field stars, called the Interface sequence, can be fit by a function (as we do below), giving the mass dependence of stellar rotation.

3. (Re-)determination of the mass- and age dependencies

Having determined that rotation has both mass- and age dependencies, how is one to specify them independently using one set of data, and without greatly compromising the determined dependencies? One way forward is to realize that open clusters can specify the mass dependence regardless of whether or not we make some error in their ages - after all, they are all clustered near ZAMS ages - while the Sun provides a datum with a very well-defined age far out, but obviously no information about the mass dependence. These facts suggest the use of open clusters to decide the mass dependence, and the Sun to decide the age dependence. The effect is to follow the Copernican Principle and assume that the Sun is the perfect representative of its class of star. (We note that the same principle guides the solar calibration of the classical isochrone method.)

The construction of an appropriate fit for the mass dependence requires the removal of stars that are not on the I sequence in open clusters. This cannot yet be done unambiguously using only color-period data because the position of the I sequence has yet to be specified well. That is part of the goal of this paper. For clusters where X-ray data are also available, we get a additional handle on classifying these stars using the correspondence noted in Barnes (2003b). There, the classification in X-rays of unsaturated, saturated, and super-saturated stars is shown to correspond, on a star-by-star basis with I-, g-, and C stars respectively. We therefore select the unsaturated stars, which are all I sequence stars in the color-period diagram, and for each cluster, we plot $P/\sqrt{Cluster\ Age}$ against $(B-V)_0$. These stars define a sequence in color-$P/\sqrt{Age}$ space, and we can now discard stars from the other clusters without X-ray information that lie far away from this sequence. The aim is to do this conservatively, so as to retain as many stars as possible for a proper definition of the I sequence, while removing clear C-, g-, or alias period stars. While it is true that this determination is done subjectively at present, it is done as empirically as we possibly can at the present time[15]. The remaining stars happen to lie near the trial function $f(B-V)$ that was used in Barnes (2003a), but this function does not obviously pre-determine the new one.

[14]This formula yields ages in close agreement with those for old stars calculated using the formulae in Soderblom, Duncan & Johnson (1991), but is generally considered to be an improvement for young stars because saturation effects are taken into account [cf. Barnes 2001].

[15]Judgements such as these are routinely made during classical isochrone fitting. A rich dataset or two, such as the one for M35 (S. Meibom, in prep) should eliminate much of the ambiguity within a year or two.

This exercise suggests that slight modifications to the open cluster ages are needed to tighten the overlap of the individual I sequences. We have made these slight adjustments in order to ensure a valid result for the mass dependence. The ages used are: IC 4665: 40Myr; Alpha Per: 110Myr; Pleiades: 120Myr; NGC 2516: 180Myr; M 34: 200Myr; NGC 3532: 250Myr; Coma Ber: 600Myr; Hyades: 600Myr. We have not used IC 2391 and IC 2602 because although they possess identifiable sequences, they are some distance off the sequence defined by the other clusters, a fact we attribute to the residual effects of pre-main sequence evolution[16]. These minor age adjustments are justifiable because, in any case, we are not using the open clusters to decide the age dependence of rotation. We are effectively merely using them to set the "zero-point" of the age dependence. We know that their I sequence age dependence is roughly Skumanich-style.

Having removed the non-I sequence stars, we note a tight mass-dependence for which we desire a functional form. The trial function $f(B-V) = \sqrt{(B-V-0.5)} - 0.15(B-V-0.5)$ has an undesirable singularity at $(B-V) = 0.5$, which we would like to move blueward, to accommodate the late F stars. We would also like to retain an analytic function. A function of the form $f(B-V) = a.(B-V-0.4)^b$ where a and b are fitted constants, seems to be appropriate (and will permit appropriate error analysis later). Using the R statistics package (Ihaka & Gentleman, 1996) to do the fit, we get $a = 0.7725 \pm 0.011$ and $b = 0.601 \pm 0.024$. This function is plotted with a solid line in Fig. 5 over the I sequence stars in the open clusters listed above. The standard error on the residuals is 0.0795 on 182 degrees of freedom.

To show that the fit is appropriate, we also display using a dashed line in Fig. 5 the result of fitting a non-parametric trend curve using the function *lowess* in the R statistics package[17]. The close correspondence between the two curves shows that the function chosen above is appropriate for these data.

Having determined the mass-dependence using open clusters, we check that it is appropriate for the field stars, which provided the motivation for improving the representation of $f(B-V)$. We plot the new and old dependencies in Fig. 6, over the the Mt. Wilson stars (same data as in Fig. 4), and again *assuming that the chromospheric ages are correct.* The figure displays the difference between the old and new functions, f, in relation to the Mt. Wilson stars. (This discrepancy between f and the F star data is partially attributable to the assumption of correct chromospheric ages for blue stars and is addressed in another section below.)

Having specified the mass dependence using open clusters, and having shown that the Sun and field stars also follow this mass dependence, we can now determine the age dependence. We know that the age dependence $g(t)$ will roughly be $\sqrt{t}$, but the open clusters are too young to be effective calibrators, nor are their ages known to sufficient precision. In contrast, the rotation rate of the Sun is perhaps the most fundamental datum in stellar rotation, and its parameters are the fundamental calibrators for theoretical stellar models. In keeping with this tradition, (and older ones of calibrating clocks by the sun), we choose to specify the age dependence via a solar calibration. Representing the age dependence using $g(t) = t^n$, and calibrating the index n using the Sun's measured mean rotation period of 26.09d (Donahue et al. 1996), a solar $B-V$ color of 0.642 (Holmberg et al. 2005) and a solar age of 4.566 Gyr (Allegre et al. 1995) yields $n = 0.5189 \pm 0.0070$, where the error on n has been calculated by simply propagating the errors on the other terms and assuming 1 d and 50 Myr errors in the period and age of the sun respectively. This calculation is

[16]The 110 Myr age for Alpha Per might also be a surprise to some. In fact, we guess that the underlying rotational behavior might also originate in residual effects from pre-main sequence evolution, similar to IC 2391 & IC 2602. However, we have chosen to retain it in this analysis because we cannot yet afford to lose the many periods in this cluster (contributed by Prosser & Grankin 1997).

[17]The *lowess* function implements a locally weighted regression smoothing procedure using a polynomial. No significant difference is seen with other smoothing procedures.

detailed in the appendix to this paper.

So, the final result works out to be: $P(B-V,t) = f(B-V).g(t)$, where

$$f(B-V) = (0.7725 \pm 0.011) \times (B - V_0 - 0.4)^{0.601 \pm 0.024} \tag{1}$$

and

$$g(t) = t^{0.5189 \pm 0.0070} \tag{2}$$

a result which is simultaneously analytical, simple, separable, almost Skumanich, fits the mass dependence of the open clusters, and the age dependence specified by the Sun.

We know that for the I sequence stars, whether in clusters or in the field, the rotation rate is given by $P(B-V,t) = f(B-V).g(t)$, where $f(B-V)$ and $g(t)$ were determined above. This is true for each star. Therefore $t = g^{-1}[P(B-V,t)/f(B-V)]$. Explicitly,

$$log(t_{gyro}) = \frac{1}{n}\{log(P) - log(a) - b \times log(B - V - 0.4)\} \tag{3}$$

where t is in Myr, $B-V$ and P are the measured color and rotation period (in days) respectively, $n = 0.5189 \pm 0.007$, $a = 0.7725 \pm 0.011$, and $b = 0.601 \pm 0.024$.

4. Errors in the ages

The ages from gyrochronology become truly useful only when we can estimate their errors and show that they are acceptable. A crude estimate of the error is simply the spread in the function $f(B-V)$.

$$\frac{\delta t}{t} = \frac{1}{n}\frac{\delta f}{f} \approx 2\frac{\delta f}{f} \tag{4}$$

An estimate for δf is the standard error of the residuals from the fit to f, which we have derived using R, and which is 0.0795 on 182 degrees of freedom. (f itself as shown above, is of course known much better because of the number of points involved.) For a K star, at $B-V = 1$, roughly the middle of our distribution, $f = 0.57$, so that

$$\frac{\delta t}{t} \approx 28\% \tag{5}$$

The errors in f are heteroscedastic, as can be seen from Fig. 5, and on the reasonable assumption that they scale with f, we can simply adopt this value of 28% error in the ages for all G, K, & early M stars. This gives a representative number, but a uniform adoption of this error overestimates the age error for our stars[18], and masks the underlying variations, which we elucidate below.

4.1. Derivation of errors

We begin with the representation

$$P = f(B-V).g(t) \tag{6}$$

[18]This can be traced to the generous limits adopted in the present instance among the open clusters for inclusion as an I sequence star, resulting in considerable contamination from incorrect rotation periods, g- and perhaps even C stars. This contamination results in a large scatter in f, but f itself is defined much better because of the large number of data points involved. Further work is needed in open clusters to clarify this matter.

where $P, B-V$, and t are the period, color and age of the star respectively, and f and g are the color and age dependencies, as before. Taking logs and differentiating, we get

$$\frac{dP}{P} = \frac{df}{f} + \frac{dg}{g} \tag{7}$$

Now, $g(t) = t^n$, so $dg/g = ndt/t + ln\, t\, dn$, where $n \approx 0.5$. Thus,

$$\frac{dP}{P} = \frac{df}{f} + n\frac{dt}{t} + ln\, t\, dn \tag{8}$$

Now, $f(B-V) = ax^b$, where $x = B - V - 0.4$ and a and b are fitted constants (with associated errors). Differentiating, $df/f = da/a + b\, dx/x + ln\, x\, db$. Thus,

$$\frac{dP}{P} = \frac{da}{a} + b\frac{dx}{x} + ln\, x\, db + n\frac{dt}{t} + ln\, t\, dn \tag{9}$$

Substituting, re-arranging, and adding the errors in quadrature under the usual assumption of independence yields

$$(n\frac{\delta t}{t})^2 = (ln\, t\, \delta n)^2 + (\frac{\delta P}{P})^2 + (\frac{\delta a}{a})^2 + (b\frac{\delta x}{x})^2 + (ln\, x\, \delta b)^2 \tag{10}$$

The one term above that requires further attention is the period (P) term. There are two contributions to the period error - the measurement error, and differential rotation, which can be added in quadrature: $(\delta P/P)^2 = (\delta P_{msrmnt}/P)^2 + (\delta P_{dffrtn}/P)^2$. The period determination itself is not usually a great contributor to the error, but the differential rotation term could potentially be a deal-breaker. Donahue et al. (1996) concluded that the dependence was a simple function of the rotation period alone, and their results (see esp. Fig. 3 there) suggest that the period range, $\Delta P = P_{max} - P_{min}$, can be represented simply by $log\,(\Delta P) = -1.25 + 1.3\, log < P >$. The long baseline of their dataset suggests that ΔP corresponds to 2σ, so that the (1σ) period error is simply a quarter of this: $log\,(\delta P_{dffrtn}) = -1.85 + 1.3\, log < P >$ so that,

$$(\frac{\delta P}{P})^2 = (\frac{\delta P_{msrmnt}}{P})^2 + (10^{-1.85}\, P^{0.3})^2 \tag{11}$$

Substituting this in equation (10) gives

$$(n\frac{\delta t}{t})^2 = (ln\, t\, \delta n)^2 + (\frac{\delta P_{msrmnt}}{P})^2 + (10^{-1.85}\, P^{0.3})^2 + (\frac{\delta a}{a})^2 + (b\frac{\delta x}{x})^2 + (ln\, x\, \delta b)^2 \tag{12}$$

Putting in some of the numerical values will allow us to understand the dependencies of the errors. From equation (2) - see appendix 1 for the details - $n = 0.0519 \pm 0.007$. The error in the period determination for the Mt. Wilson stars is 0.25%-1%, 0.5%-2%, 2%-4% for periods less than 20d, greater than 20d, or periods between 30d and 60d respectively (Donahue et al. 1996). A 1% error seems to be a reasonable representation for the samples considered here. For $\delta x = \delta(B-V)$, we adopt the value of 0.01 suggested by the precision of the datasets considered below. This might need to be increased to 0.02 for data acquired through CCD photometry (assuming independent errors of 0.015 in each filter), but we note that this error could be considerably lower for data acquired through photoelectric photometry. From section 3 and equation (1), $a = 0.7725 \pm 0.011$ and $b = 0.601 \pm 0.024$. We input these values to get (in the same order as above)

$$(n\frac{\delta t}{t})^2 = (0.007\, ln\, t)^2 + (0.01)^2 + (0.014\, P^{0.3})^2 + (\frac{0.011}{0.7725})^2 + (0.6\frac{0.01}{x})^2 + (0.024\, ln\, x)^2 \tag{13}$$

or

$$(n\frac{\delta t}{t})^2 = 10^{-4}[(0.7\,ln\,t)^2 + (1)^2 + (1.4\,P^{0.3})^2 + (1.424)^2 + (\frac{0.6}{x})^2 + (2.4\,ln\,x)^2] \tag{14}$$

or

$$(n\frac{\delta t}{t})^2 = 10^{-4}[\frac{1}{2}(ln\,t)^2 + 1 + (1.4\,P^{0.3})^2 + 2 + (\frac{0.6}{x})^2 + (2.4\,ln\,x)^2] \tag{15}$$

Thus,

$$\frac{\delta t}{t} = 2\% \times \sqrt{3 + \frac{1}{2}(ln\,t)^2 + 2\,P^{0.6} + (\frac{0.6}{x})^2 + (2.4\,ln\,x)^2} \tag{16}$$

which shows that the age error is always greater than $\sim 11\%$ for conditions similar to those assumed here. (Recall that t is in Myr, P is in days, and $x = B - V_0 - 0.4$.) For 1 Gyr-old stars of spectral types late F, early G, mid K and early M respectively, we get

$$\frac{\delta t}{t} = 2\% \times \begin{cases} \sqrt{26.9 + 6.4 + 66.5} & \text{when } B-V = 0.5\ (P = 7d); \\ \sqrt{26.9 + 8.9 + 16.9} & \text{when } B-V = 0.65\ (P = 12d); \\ \sqrt{26.9 + 12.1 + 2.5} & \text{when } B-V = 1.0\ (P = 20d); \\ \sqrt{26.9 + 15.4 + 0.35} & \text{when } B-V = 1.5\ (P = 30d). \end{cases} \tag{17}$$

which shows the relative contributions of the period and color errors (second and third terms, respectively), or,

$$\frac{\delta t}{t} = \begin{cases} 20\% & \text{when } B-V = 0.5\ ; \\ 15\% & \text{when } B-V = 0.65\ ; \\ 13\% & \text{when } B-V = 1.0\ ; \\ 13\% & \text{when } B-V = 1.5\ . \end{cases} \tag{18}$$

The behaviors of the function f and of differential rotation are such that the color and period errors dominate for blue and red stars respectively to give a total error of $\sim$15%. The errors calculated using equation (16) are the ones quoted for the gyrochronology ages in the remainder of this paper. Setting the P and $B - V$ errors equal leads to a trancendental equation which separates the color-period space into two regions, a blue one where color errors dominate, and a red one where the period errors (mostly differential rotation) dominate. The separator is a steep function in color-period space, and is roughly at solar color.

How well these errors represent the true errors of this technique future work will show. We simply note here that the very possibility of calculating the errors distinguishes gyrochronology from other stellar chronometric methods.

5. Comparison with the chromospheric clock

Before calculating gyro ages for stars where other ages are not available, it is necessary to consider whether these ages agree at least roughly with others that might be available. We stress here that the ages derived through gyrochronology in this paper are *independent of other techniques*, except for the calibration using the Sun, whose age is determined using radioactivity in meteorites (see prior section). In particular, these ages are independent of chromospheric and isochrone ages, except for the common solar calibration point[19].

Potentially, the best way to test these ages would be to derive ages for open clusters where other ages are also available. This is not possible in this work because the open clusters have been used here to derive

[19]The calibration issue is discussed later in this section.

the mass dependence of gyrochronology, and this required a prior knowledge of their ages. Additional data will allow such a test in the future[20]. Isochrone ages for main sequence field stars are not reliable enough to serve as a test. (Section 9 elaborates on this.) What is possible is a test against chromospheric ages for field stars.

Despite the obviously large errors associated with the method (see below), chromospheric ages have thus far been considered to be the best ones available at present for single field stars. Furthermore, there exists a substantial and uniform sample, the Mt. Wilson stars, for which the chromospheric emission is known very well (over decades), for which the chromospheric ages are believed to be relatively secure, and for which *measured* rotation periods are also available[21]. These facts allow us to compare the ages from the two (independent) techniques below.

5.1. How is the chromospheric age of a star calculated?

There has been considerable work on the determination of the rate of decay of chromospheric emission with age since the results of Skumanich (1972). The two sources generally quoted for a relationship between chromospheric age and R'_{HK} are Donahue (1998) and Soderblom et al. (1991). Although we will end up using the former to calculate chromospheric age, it is necessary to discuss both to understand the relevant issues. The key feature of both relationships is that once the measurement of stellar chromospheric emission has been made (repeatedly or not), it can immediately be converted into an age, without additional information. (Wright et al. 2004 have shown subsequently that stars previously considered to be in Maunder Minima are in fact somewhat evolved, so caution is advisable with respect to the basic properties of the star.)

5.1.1. The Donahue (1998) relationship

The relationship given in Donahue (1998) is:

$$log[t_{Chromospheric}] = 10.725 - 1.334R_5 + 0.4085R_5^2 - 0.0522R_5^3 \tag{19}$$

where $R_5 = 10^5 R'_{HK}$ and the age, t, is measured in Gyr. This relationship is essentially identical to the one in Soderblom et al (1991) (discussed below) for ages greater than 1 Gyr. The deviation between the two relationships pertains to younger stars, including those in the Hyades, Coma, Ursa Major, Pleiades, and NGC 2264 open clusters, for which it claims a better age calibration. This particular feature has prompted us to use it instead of the Soderblom et al. (1991) relationship.

However, it does have two serious limitations: Firstly, it does not provide errors on the ages so derived. (However, Donahue (1998) does list the discrepancies in chromospheric age for a number of wide binaries and triple systems. The mean discrepancy for the systems listed is 0.85 Gyr on a mean age of 1.85 Gyr, which suggests a fractional age error of ∼46%, in rough agreement with the errors quoted in Soderblom et al. 1991.) Secondly, there are no refereed publications that spell out the details of the derivation. Nevertheless,

[20]In fact, two are underway using new data in M 34 (James et al. 2007) and M 35 (Meibom et al., in prep.)

[21]There exists another sample of 19 Southern stars for which chromospheric emission (from Henry et al. 1996) and rotation periods are both available. These overlap with another sample of stars discussed here in section 6 below, and the corresponding comparison is presented there.

it has been used by Wright et al. (2004) to derive ages for stars in the sample being studied by the Marcy group for evidence of planets, and we follow suit.

5.1.2. *The Soderblom et al. (1991) relationship*

The relationship between chromospheric emission and age provided in Soderblom et al. (1991) is

$$log[t_{Chromospheric}] = (-1.50 \pm 0.003) log R'_{HK} + (2.25 \pm 0.12) \tag{20}$$

where the age, t, is again in Gyr. (*Note especially that errors are provided.*) This expression, equation (3) from Soderblom et al. (1991), is based on 42 data points and three "fundamental" points, which are the Sun, the Hyades, and the Ursa Major Group. This relationship passes through the data point for the Sun, using the value of $\mathrm{Log}\, R'_{HK} = -4.96$, quoted there, and a solar age of 4.6 Gyr. It is equivalent to $R'_{HK} \propto t^{-2/3}$, and the authors note that a case could be made for using slightly different relationships, including one where $R'_{HK} \propto t^{-3/4}$, equation (2) in their paper, depending on the choice of data points included. This relationship has a standard deviation of 0.17 dex, which corresponds to an error of ∼40%. In the absence of errors for the Donahue (1998) relationship above, we simply adopt this value of 0.17 dex for the error in chromospheric ages calculated using that relationship also.

We note also that the Soderblom et al. (1991) relationship is in some ways the culmination of an extensive and self-consistent study by Soderblom and collaborators, and is explained in detail in a series of papers, including Duncan (1984), Duncan et al. (1984), Soderblom (1985), and Soderblom & Clements (1987). The reader is referred to these for the technical details, and especially for the overall logic of the scheme.

One point about the calibration of the technique needs to be mentioned, because it also relates to the calibration of gyrochronology. The ages against which the above relationship is calculated are derived using isochrone fits to visual binary stars, and to the "fundamental" points, which are again based on isochrone fits. The entire isochrone technique itself is calibrated by ensuring that the appropriate solar model matches the solar parameters, usually the radius and the luminosity, at solar age. Thus, this technique is also ultimately calibrated on the Sun.

5.2. Comparison between chromospheric and gyro ages for the Mt. Wilson stars

We use the data compilation published in Baliunas et al. (1996) and Noyes et al. (1984) for the Mt. Wilson stars and calculate the chromospheric ages using the formula in Donahue (1998). The chromospheric ages for the Mt. Wilson stars, calculated using the above formula, are listed in Table 3. For obvious reasons, stars with calculated periods have been excised, and only stars with measured periods (71 in number) have been retained for this comparison[22].

For the same stars, we can calculate ages via gyrochronology using equation (3) from Section 3 above. These ages are calculated and listed in Table 3. The errors on these ages, calculated using equation (16) from section 4, are also listed in the table.

[22]We have also had to eliminate HD124570, which, although not considered evolved in the Mt. Wilson datasets, is now known to be so (e.g. Cowley, 1976; SAB thanks Brian Skiff for researching this star).

The gyro ages are plotted against the chromospheric ages for the same stars in Fig. 7, with small green and large red symbols marking the young (Y) and old (O) Mt. Wilson stars, as classified by Vaughan (1980). Note that both techniques segregate the Y and O stars. The demarcation is sharper in chromospheric age, as it ought to be, since this is the criterion chosen to classify the stars as young or old. The figure shows that, apart from a slight tendency towards shorter gyro ages (discussed further below), there is general agreement between the chromospheric- and gyro ages for this sample. Note that except for a few stars discussed below, there are no stars with widely discrepant ages, unlike the corresponding comparison with isochrone ages, where discrepancies are routine (c.f. Fig. 2 in Barnes 2001).

Note especially that the gyro ages are well-behaved for every single star here, ranging from just under 100 Myr to just over 10 Gyr. In contrast, there are three stars whose chromospheric ages are almost certainly incorrect (see table 4). The two stars HD82443 and HD129333 have chromospheric ages of 0.67 Myr and 0.002 Myr respectively. These stars are undoubtedly young but the interpretation of these numbers eludes us. The corresponding gyro ages for these stars are 164 ± 18 Myr and 73 ± 9 Myr, which suggest that they are essentially on the Zero Age Main Sequence (ZAMS). Also, for one star, HD95735, the chromospheric age is 20 Gyr, greater than that of the universe (dotted lines). This cannot be correct. The gyro age for this star is 3.2 ± 0.5 Gyr, definitely younger than the Sun. At least in this restricted sense and for specific stars, the gyro ages are better defined than chromospheric ages.

Fig. 8 elaborates on the difference between the chromospheric- and gyro ages. The solid line again denotes equality, while the dashed line, at $t_{gyro} = 0.74 \times t_{chrom}$, bisects the data points. This shows that the gyro ages are roughly 25% lower than the chromospheric ages overall. The nature of the disagreement can be probed by segregating the stars by color. Thus, stars bluer than $B - V = 0.6$ and redder than $B - V = 0.8$ are plotted using blue crosses and red asterisks respectively, while those with intermediate colors are plotted using green squares. This exercise shows that there is good agreement for stars redward of $B - V = 0.6$ and that the above discrepancy pertains only to the blue stars.

5.3. Discussion of the disagreement between the techniques

This disagreement can be probed further than merely stating that the errors in the chromospheric ages are greater than those of the gyro ages. It must originate in either the gyro ages for blue stars being systematically shorter or the chromospheric ages for these being systematically longer, or both. The discussion below, and the results of testing binaries, performed in Section 8, suggest that the chromospheric ages are the more problematical.

With respect to the gyro ages, one defect is that the open cluster sample used to define the mass dependence of rotation does not contain stars with $B - V$ colors blueward of 0.5 because it is not yet possible to distinguish between very blue C- and I-type stars. This means that $f(B-V)$ is an extrapolation for stars with $0.4 < B - V < 0.5$. The fitting function $f(B - V)$, blueward of $B - V = 0.6$, appears to be somewhat elevated with respect to the data points displayed in Fig. 5. This would tend to lower the gyro ages. If $f(B - V)$ were lowered in this region by ~20%, the gyro ages would be raised by a factor of ~1.5, which is doable considering the main sequence lifetime of the F stars, but a reduction of ~30% would double the gyro ages, and might run afoul of standard stellar evolution, because the main sequence lifetime of a late-F star is ~5 Gyr.

The chromospheric ages are not blamefree in this regard either, and it is almost certain that they have

been overestimated for F stars[23]. Four F stars have chromospheric ages in excess of 7 Gyr and one in excess of 6 Gyr. These values exceed the main sequence lifetime of a late-F star, which is 5 Gyr. In comparison, all of these five F stars have shorter gyro ages, with the oldest of them assigned a gyro age of 2.3 Gyr. These stars are also listed in Table 4. These stars are located at higher chromospheric ages than 5 Gyr, marked in Fig. 8 with thin dashed blue lines. Thus, while the gyro ages have possibly been slightly underestimated for F stars, it is almost certain that the corresponding chromospheric ages have been overestimated.

5.4. Additional issues with chromospheric ages

The derivation of a chromospheric age for a star is complicated by the natural variability of chromospheric emission with stellar rotational phase and stellar cycle (e.g. Wilson, 1963). In fact, rotation-related variations in chromospheric emission are the preferred way of deriving rotation periods for old stars. Binarity or other effects could result in additional variability. These variations make it necessary for repeated measurement on a suitable timescale of the chromospheric emission from a star to ensure that the measured average is a good representation of the chromospheric emission at that age for the star. Therefore, it is unlikely that one can make a single measurement of chromospheric emission and derive a good age for a star.

Some of the issues with chromospheric ages are illustrated by the recent work of Giampapa et al. (2006) on the chromospheric properties of the sun-like stars in the open cluster M 67, averaged over several seasons of observing. This cluster is known to be ∼4 Gyr old (e.g. VandenBerg & Stetson, 2004). There is no evidence that the stars in this cluster are not coeval.

Correspondingly, Giampapa et al. (2006) derive mean and median ages in the range 3.8 Gyr to 4.3 Gyr. What is surprising is that the chromospheric ages for individual stars range from under 1 Gyr to 7.5 Gyr (see Fig. 13 in their paper). Admittedly, the vast majority of the stars have chromospheric ages between 2 Gyr and 6 Gyr, but this range is not small either. This result seems to cast doubts on the precision in ages for single stars obtainable even in principle with chromospheric emission because measuring the age for M 67 using a random cluster member could result in such large age variability.

[23]The embedded mass dependence in the chromospheric ages can be traced to Noyes et al. (1984), where the mass dependence of chromospheric emission was based on the Rossby Number, and theoretical estimates of the variation of convective turnover timescale with stellar mass. The residual mass dependence could be removed eventually with the availability of larger samples of stars, especially those in open clusters.

Table 3. Gyrochronology ages and errors for the Mt. Wilson stars.

HD	$B-V$	P_{rot}(d)[A]	$-log < R'_{HK} >$	t_{chrom}/Myr	t_{iso}/Myr[B]	t_{gyro}/Myr	δt_{gyro}/Myr
Sun	0.642	26.09[C]	4.901	3895	......	4566	770
1835	0.66	7.78	4.443	601	<1760	408	54
2454	0.43	3	4.792	2609	......	790	350
3229	0.44	2	4.583	1251	......	260	91
3651	0.85	44	4.991	5411	>11800	6100	990
4628	0.88	38.5	4.852	3250	>6840	4370	680
6920	0.60	13.1	4.793	2618	......	1510	240
10476	0.84	35.2	4.912	4056	>8840	4070	630
10700	0.72	34	4.958	4802	>12120	5500	910
10780	0.81	23	4.681	1764	10120	1945	280
16160	0.98	48.0	4.958	4802	540	5370	850
16673	0.52	7	4.664	1664	......	820	150
17925	0.87	6.76	4.311	74	<1200	157	16
18256	0.43	3	4.722	2033	......	790	350
20630	0.68	9.24	4.420	489	<2760	522	69
22049	0.88	11.68	4.455	659	<600	439	52
25998	0.46	2.6	4.401	398	......	270	70
26913	0.70	7.15	4.391	352	......	294	36
26965	0.82	43	4.872	3499	>9280	6320	1030
30495	0.63	7.6[D]	4.511	923	6080	450	62
35296	0.53	3.56	4.378	294	......	202	33
37394	0.84	11	4.454	654	<1360	432	52
39587	0.59	5.36	4.426	518	4320	286	41
45067	0.56	8	5.094	7733	5120	760	120
72905	0.62	4.69	4.375	281	......	187	24
75332	0.49	4	4.464	703	1880	387	79
76151	0.67	15	4.659	1635	1320	1380	200
78366	0.60	9.67	4.608	1370	<680	840	130
81809	0.64	40.2	4.921	4193	......	10600[E]	1900
82443	0.77	6	4.211	0.7	......	164	18
89744	0.54	9	5.120	8421	1880	1110	190
95735	1.51	53	5.451	20028	......	3070	460
97334	0.61	8	4.422	499	<2920	551	80
100180	0.57	14	4.922	4209	3800	2070	350
101501	0.72	16.68	4.546	1082	>11320	1400	200
106516	0.46	6.91	4.651	1591	......	1770	480
107213	0.50	9	5.103	7966	2040	1630	330
114378	0.45	3.02	4.530	1010	......	445	130
114710	0.57	12.35	4.745	2205	<1120	1630	270
115043	0.60	6	4.428	528	......	335	47
115383	0.58	3.33	4.443	601	<760	122	17
115404	0.93	18.47	4.480	779	......	950	120
115617	0.71	29	5.001	5609	8960	4200	680
120136	0.48	4	4.731	2098	1640	443	97
129333	0.61	2.80	4.152	0.002	<1440	73	9
131156A	0.76	6.31[F]	4.363	232	<760	187	21
131156B	1.17	11.94[G]	4.424	508	>12600	265	28
141004	0.60	25.8	5.004	5669	6320	5570	990
143761	0.60	17	5.039	6413	9720	2490	410
149661	0.82	21.07	4.583	1251	<4160	1600	220
152391	0.76	11.43	4.448	625	720	587	75
154417	0.57	7.78	4.533	1023	4200	670	105
155885	0.86	21.11[H]	4.559	1141	......	1440	200
155886	0.86	20.69[I]	4.570	1191	......	1390	190
156026	1.16	18.0[J]	4.622	1439	<480	593	71

Table 3—Continued

HD	$B-V$	P_{rot}(d)[A]	$-log < R'_{HK} >$	t_{chrom}/Myr	t_{iso}/Myr[B]	t_{gyro}/Myr	δt_{gyro}/Myr
160346	0.96	36.4	4.795	2637	......	3280	490
165341A[K]	0.86	20	4.548	1091	......	1300	180
166620	0.87	42.4	4.955	4750	>11200	5400	860
178428	0.70	22	5.048	6616	......	2560	390
185144	0.80	27	4.832	3019	......	2730	410
187691	0.55	10	5.026	6128	3200	1250	210
190007	1.17	28.95	4.692	1832	<1760	1460	200
190406	0.61	13.94	4.797	2657	3160	1610	250
194012	0.51	7	4.720	2019	......	900	170
201091	1.18	35.37	4.764	2359	<440	2120	300
201092	1.37	37.84[L]	4.891	3753	<680	1870	260
206860	0.59	4.86	4.416	470	<880	237	33
207978	0.42	3	4.890	3740	......	1270	810
212754	0.52	12	5.073	7207	......	2300	440
219834B[M]	0.91	43	4.944	4563	>13200	5040	800
224930	0.67	33	4.875	3538	......	6330	1080

[A]Only *measured* periods for unevolved stars are listed. They are taken, in order of priority, from Donahue, Saar & Baliunas (1996), Baliunas et al. (1983), and Baliunas, Sokoloff & Soon (1996). The first of these lists the average rotation period of several seasonal periods (and the differential rotation), hence the priority assigned to this paper, the second a single best period determined from an intensive chromospheric monitoring program in 1980-81 (with the error of that single determination), and the third a mean rotation period (to lower precision than the previous two publications) based on the entire extant intensive sampling database.

[B]The isochrone ages listed in this and subsequent tables are taken from Takeda et al. (2007).

[C]The mean solar period of 26.09d, taken from Donahue et al. (1996), represents the average of 8 determinations, and is presumably representative of the mean latitude of sunspot persistence, while the ∼25d period usually listed is the mean equatorial rotation period.

[D]Baliunas et al. (1996) list a significantly different period of 11d.

[E]The gyro age should be treated with caution because this star is a spectroscopic binary (Pourbaix, 2000).

[F]Period is from Donahue et al. (1996). Baliunas et al. (1996) simply list a period of 6d.

[G]Period is from Donahue et al. (1996). Baliunas et al. (1996) list a period of 11d.

[H]This period is from Donahue et al. (1996). Baliunas et al. (1983) list a very similar period of 22.9±0.5d.

[I]This period is from Donahue et al. (1996). Baliunas et al. (1983) list a very similar period of 20.3±0.4d.

[J]This period is from Baliunas et al. (1983). Baliunas et al. (1996) list a period of 21d for all 3 components HD155885, HD155886, & HD156026.

[K]The second component, HD165341B, of this binary is also in the Mt. Wilson sample (e.g. Baliunas et al. 1996), but the period of 34d is one calculated from chromospheric emission.

[L]The periods listed for HD201091 and HD201092 are from Donahue et al. (1996). Baliunas et al. (1996) list similar periods of 35 and 38d respectively, while Baliunas et al. (1983) list the somewhat discrepant periods of 37.9±1.0 and 48d respectively.

[M]The other component in this system, HD219834A, is also in the Mt. Wilson dataset, but it seems to be evolved, and so is excluded here.

6. Ages for young field stars from the Vienna-KPNO (Strassmeier et al. 2000) survey

Another group of stars amenable to the calculation of ages via gyrochronology is the field star sample of Strassmeier et al. (2000). Unlike the older Mt. Wilson star sample, this group contains some stars for which gyro ages are not yet appropriate, and here we demonstrate how to identify and excise these stars and calculate ages for the rest.

The full Strassmeier et al. (2000) sample consists of 1058 Hipparcos stars with various measured parameters, including chromospheric emission. Of these 1058 stars, 140 have measured rotation periods, and of these, we are interested here only in stars on the main sequence. Using the Luminosity classes supplied by Strassmeier et al., we have simply selected the dwarf stars, and excised the others. This leaves us with 101 dwarf stars with measured rotation periods.

These 101 stars with measured periods are plotted in a color-period diagram in Fig. 9. We superimpose an I sequence curve corresponding to 100 Myr, and assume that the 16 stars below this curve are C- or g stars, while those above are I sequence stars similar to those in the Mt. Wilson sample. In the scenario from Barnes (2003a), the stars below are either on the C sequence appropriate to their age, or in the transition, g, between the C- and I sequences. This cut is undoubtedly conservative, since there are open clusters younger than 100 Myr known to possess I sequences, but we prefer to lose a few stars rather than risk over-extending the technique. This leaves us with 85 potential I sequence stars amenable to gyrochronology.

These 85 I sequence stars are again plotted in Fig. 10, where now we have superimposed isochrones corresponding to ages of 100-, 200-, & 450 Myr, and 1-, 2-, & 4.5 Gyr. We see that the Strassmeier et al. (2000) main sequence sample with measured periods consists mainly of stars younger than 1 Gyr, and all but 4 younger than 2 Gyr. In fact the median age for the sample is 365 Myr, in keeping with the selection of this sample for activity.

Gyro ages are calculated as above for each star, and listed in Table 5, along with their basic measured properties. Almost all of these stars are redder than the Sun. For such stars, as shown in the previous section, there is very good agreement between gyro- and chromospheric ages, and consequently, some confidence can be attached to the calculated ages. We have also calculated the errors on these ages, using equation (16) from section 4, and listed these in the final column of the table.

At present, no good test of these ages is possible. Although the chromospheric emission has been measured, the measurements have been made with a small telescope, and are not long-term averages, so that the values quoted cannot be treated with the confidence associated, for instance, with the Mt. Wilson measurements, and there is considerable scatter, as the few repeat measurements demonstrate. Furthermore,

Table 4. Stars with suspect chromospheric ages.

Star	$B-V$	Age_{chromo}	Age_{gyro}	Comment
HD45067	0.56	7.73 Gyr	0.76±0.1 Gyr	Chromo age > lifetime
HD82443	0.77	0.7 Myr	164±18 Myr	Chromo age too small?
HD89744	0.54	8.42 Gyr	1.11±0.19 Gyr	Chromo age > lifetime
HD95735	1.51	20 Gyr	3.1±0.46 Gyr	Chromo age > Age of universe
HD107213	0.50	7.97 Gyr	1.63±0.33 Gyr	Chromo age > lifetime
HD129333	0.61	0.002 Myr	73±9 Myr	Chromo age too small?
HD187691	0.55	6.13 Gyr	1.25±0.21 Gyr	Chromo age > lifetime
HD212754	0.52	7.21 Gyr	2.3±0.44 Gyr	Chromo age > lifetime

no photospheric correction has been performed, so they are on a different scale, and the relationships of Soderblom et al. (1991) and Donahue (1998) do not apply. However, it is possible to plot the R_{HK} values provided against the gyro ages calculated above to make sure that gross errors are absent, and we perform this exercise in Fig. 11. The figure demonstrates that, as expected, the chromospheric activity declines steadily with stellar age, and thus, that the gyro ages are reasonable.

Table 5. Gyrochronology ages and errors for the Vienna-KPNO survey (Strassmeier et al. 2000) stars.

HD	$B-V$	P_{rot}(d)	R_{HK}	t_{iso}/Myr	t_{gyro}/Myr	δt_{gyro}/Myr
HD691	0.76	6.105	7.2E-5	<1040	175	20
HD5996	0.76	12.165	6.0E-5	……	662	86
HD6963	0.73	20.27	4.9E-5	2240	1960	290
HD7661	0.75	7.85	7.1E-5	……	294	35
HD8997a	0.97	10.49	3.5E-5	……	292	32
HD8997b	0.97	10.49	1.8E-5	……	292	32
HD9902b	0.65	7.41	8.9E-5	……	389	52
HD10008	0.80	7.15	6.1E-5	……	211	23
HD12786	0.83	15.78	5.6E-5	……	890	120
HD13382	0.68	8.98	6.0E-5	……	494	65
HD13507	0.67	7.60	6.6E-5	<1320	373	48
HD13531	0.70	7.52	7.0E-5	<3520	324	40
HD13579A	0.92	6.79	2.8E-5	>8840	141	14
HD16287	0.94	11.784	5.3E-5	<2360	390	45
HD17382	0.82	>50	6.1E-5	……	>8450[A]	>1400
HD18632	0.93	10.055	5.3E-5	>7800	293	33
HD18955a	0.86	8.05	6.1E-5	……	225	25
HD19668	0.81	5.41	4.1E-5	……	120	12
HD19902	0.73	>50	5.4E-5	……	>11200	>2000
HD20678	0.73	5.95	6.6E-5	……	185	21
HD27149a	0.68	8.968	5.7E-5	……	492	65
HD27149b	0.68	8.968	5.1E-5	……	492	65
HD28495	0.76	7.604	9.3E-5	……	268	31
HD31000	0.75	7.878	8.2E-5	……	296	35
HD53157	0.81	10.88	6.2E-5	……	460	56
HD59747	0.86	8.03	6.5E-5	<920	224	24
HD73322	0.91	16.41	5.1E-5	……	788	100
HD75935	0.77	8.19	6.3E-5	……	299	35
HD77825	0.96	8.64	5.2E-5	……	205	22
HD79969	0.99	43.4	3.6E-5	……	4340	670
HD82443	0.78	5.409	9.4E-5	……	130	14
HD83983	0.88	10.92	3.9E-5	……	386	45
HD87424	0.89	10.74	5.3E-5	<1720	365	42
HD88638	0.77	4.935	8.4E-5	……	113	12
HD92945	0.87	13.47	7.5E-5	<1280	592	73
HD93811	0.94	8.47	5.0E-5	……	206	22
HD94765	0.92	11.43	4.6E-5	<1480	384	44
HD95188	0.76	7.019	7.1E-5	<960	230	26
HD95724	0.94	11.53	5.2E-5	……	374	43
HD95743	0.97	10.33	4.0E-5	……	284	31
HD101206	0.98	10.84	4.1E-5	……	305	34
HD103720	0.95	17.16	4.2E-5	……	787	99
HD105963A	0.88	7.44	6.6E-5	……	184	19
HD105963B	0.88	7.44	6.2E-5	……	184	19
HD109011a	0.94	8.31	5.8E-5	……	199	21
HD109647	0.95	8.73	5.3E-5	……	214	23
HD110463	0.96	11.75	4.5E-5	……	371	42
HD111813	0.89	7.74	6.5E-5	……	194	21
HD113449	0.85	6.47	6.9E-5	……	152	16
HD125874	0.88	7.52	6.1E-5	……	188	20
HD128311	0.97	11.54	4.5E-5	<960	351	40
HD130307	0.89	21.79	3.9E-5	1520	1425	190
HD139194	0.87	9.37	3.8E-5	……	294	33
HD139837	0.73	6.98	8.0E-5	……	251	30
HD141272	0.80	14.045	6.7E-5	……	773	100

7. Ages for young- and intermediate-age field stars from Pizzolato et al. (2003) with X-ray measurements

There exists another comparably large group of main sequence field stars for which rotation periods and other relevant information are available. This group has been assembled by Pizzolato et al. (2003) in connection with a study of X-ray activity. There are 110 stars in this group. We remove 2 of these, HD82885 and HD136202, suspected to be evolved, leaving 108 stars. 51 of these 108 stars are in also in the Mt. Wilson sample, but the remainder do not overlap with the Strassmeier et al. (2000) stars either, and hence warrant attention. Furthermore, these stars also have measured X-ray fluxes, listed conveniently in Pizzolato et al. (2003), which allow a crude comparison with the gyro ages we derive below.

The Pizzolato et al. (2003) stars must follow the same rotational patterns as the open cluster, Mt. Wilson, and Strassmeier et al. (2000) stars. We can use the same condition that we used with the Strassmeier et al. (2003) stars to excise the C/g stars from the sample. We plot the color-period diagram for this sample in Fig. 12. Plotting a 100 Myr isochrone as before, we excise all stars below it, since these are either C- or g type stars, or only ambiguously I type. Note that of the excised stars, the ones with periods below 1 day are almost certainly C sequence stars. We also excise GL551 ($B-V = 1.90$, $P = 42d$) because it is fully convective and therefore unable to sustain an interface dynamo. (Note also that apart from this one object, there are no slow rotators redward of $B-V = 1.55$. This is consistent with the prediction by Barnes (2003a) for the terminus of the I sequence at the point of full convection.) This leaves us with 79 stars that are potentially on the I sequence in this sample.

These 79 stars are suitable for gyrochronology. We calculate the gyro ages as before, and list them, their errors, and other relevant information for these stars in Table 6. Fig. 13 displays the color-period diagram for these 79 stars, with isochrones at 100 Myr, 200 Myr, 450 Myr, 1 Gyr, 2 Gyr, 4.5 Gyr, & 10 Gyr. Fig. 13 shows that this sample spans a substantial range of ages, from 100 Myr to 6 Gyr (all but 4 of them), although most of them are younger than the Sun, and the median age for the sample is 1.2 Gyr.

These results are reasonably consistent with Sandage et al. (2003) who suggest a (classical isochrone) age for the oldest stars in the local Galactic disk of 7.4 to 7.9 Gyr ($\pm$0.7 Gyr) depending on whether or not the stellar models allow for diffusion. All the Pizzolato et al. (2003) stars except for HD81809, which is known to be a spectroscopic binary (Pourbaix, 2000), have calculated gyro ages shortward of this age.

The available X-ray data for these same stars also suggest that the gyro ages are reasonable. In Fig. 14, we plot the X-ray emission from these stars against their gyro ages. We see that the X-ray emission declines steadily, as expected, and in fact, there are no widely discrepant data points.

Finally, we note that in addition to the 51 stars in this group that are common to the Mt. Wilson sample, 19 are present in the chromospheric emission survey of Southern stars by Henry et al. (1996), where their R'_{HK} values are published. These are on the same system as the Mt. Wilson data. Thus, it is possible to compute their chromospheric ages, and compare them with the ages from gyrochronology. This comparison is shown in Fig. 15. The stars can be seen to scatter around the line of equality, and in fact, the agreement between the gyro and chromospheric ages for all but two of them is within a factor of two (see Fig. 15).

Table 5—Continued

HD	$B-V$	P_{rot}(d)	R_{HK}	t_{iso}/Myr	t_{gyro}/Myr	δt_{gyro}/Myr
HD141919	0.88	13.62	4.4E-5	......	590	72
HD142680	0.97	33.52	2.5E-5	......	2740	400
HD144872	0.96	26.02	2.7E-5	......	1720	240
HD150511	0.88	10.58	5.0E-5	......	363	42
HD153525	1.00	15.39	1.7E-5	......	577	69
HD153557	0.98	7.22	3.7E-5	......	140	14
HD161284	0.93	18.31	4.2E-5	......	930	120
HD168603	0.77	4.825	6.4E-5	......	108	11
HD173950	0.83	10.973	5.2E-5	......	442	53
HD180161	0.80	5.49	2.7E-5	......	127	13
HD180263	0.91	14.16	3.7E-5	......	593	72
HD189733	0.93	12.039	4.6E-5	......	415	48
HD192263	0.94	23.98	4.1E-5	2560	1530	210
HD198425	0.94	22.64	4.3E-5	......	1370	185
HD200560	0.97	10.526	5.2E-5	......	294	32
HD202605	0.74	13.78	5.3E-5	......	900	120
HD203030	0.75	6.664	7.1E-5	......	215	25
HD209779	0.67	10.29	6.2E-5	10000	670	92
HD210667	0.81	9.083	5.3E-5	$<$4200	325	38
HD214615AB	0.76	6.20	7.5E-5	......	181	20
HD214683	0.94	18.05	4.2E-5	......	886	110
HD220182	0.80	7.489	6.4E-5	......	230	26
HD221851	0.85	12.525	3.8E-5	......	541	66
HD258857	0.91	19.98	3.9E-5	......	1150	150
HIP36357	0.92	11.63	4.7E-5	......	397	46
HIP43422	0.75	11.14	3.3E-4	......	578	74
HIP69410	0.96	9.52	5.0E-5	......	248	27
HIP70836	0.94	21.84	3.2E-5	......	1280	170
HIP77210ab	0.83	13.83	6.3E-5	......	690	87
HIP82042	0.96	13.65	3.2E-5	......	496	59

[A] The gyro age should be treated with caution because this star is a spectroscopic binary (Latham et al. 2002).

Table 6. Gyrochronology ages and errors for the Pizzolato et al. (2003) stars.

Star	$B-V$	P_{rot}(d)	$log(L_x/L_{bol})$	t_{iso}/Myr	t_{gyro}/Myr	δt_{gyro}/Myr
Sun	0.66	25.38	-6.23	....	3980	650
GL338B	1.42	10.17	-4.90	....	140	14
GL380	1.36	11.67	-5.04	....	196	20
GL673	1.36	11.94	-4.96	....	205	21
GL685	1.45	18.60	-4.92	....	435	50
HD1835	0.66	7.70	-4.67	<1760	400	53
HD3651	0.85	48.00	-5.70	>11800	7200	1200
HD4628	0.88	38.00	-6.01	>6840	4260	660
HD10360	0.88	30.00	-5.97	<600	2700	400
HD10361	0.86	39.00	-5.94	<520	4710	740
HD10476	0.84	35.20	-6.59	>8840	4070	630
HD10700	0.72	34.50	-6.21	>12120	5660	930
HD11507	1.43	15.80	-4.76	....	324	36
HD13445	0.82	30.00	-5.56	>8480	3160	480
HD14802	0.60	9.00	-4.50	....	732	110
HD16160	0.97	45.00	-5.71	540	4840	760
HD16673	0.52	7.40	-5.03	....	910	170
HD17051	0.56	7.90	-5.02	2720	740	120
HD17925	0.87	6.60	-4.51	<1200	150	15
HD20630	0.68	9.40	-4.62	<2760	540	72
HD22049	0.88	11.30	-4.92	<600	412	48
HD25998	0.52	3.00	-4.40	....	159	27
HD26913	0.70	7.20	-4.18	....	298	37
HD26965	0.82	37.10	-5.59	>9280	4750	750
HD30495	0.64	7.60	-4.86	6080	428	58
HD32147	1.06	47.40	-5.87	<5450	4510	700
HD35296	0.53	5.00	-4.52	....	388	66
HD36435	0.78	11.20	-4.90	....	531	66
HD38392	0.94	17.30	-4.77	....	816	100
HD39587	0.59	5.20	-4.51	4320	270	38
HD42807	0.66	7.80	-4.83	....	410	54
HD43834	0.72	32.00	-6.05	8760	4900	800
HD52698	0.90	26.00	-4.74	....	1960	280
HD53143	0.81	16.40	-4.67	....	1010	130
HD72905	0.62	4.10	-4.47	....	144	18
HD75332	0.52	4.00	-4.35	1880	277	48
HD76151	0.67	15.00	-5.24	1320	1380	200
HD78366	0.60	9.70	-4.75	<680	850	130
HD81809	0.64	40.20	-6.25	....	10600[A]	1900
HD82106	1.00	13.30	-4.64	<600	435	50
HD95735	1.51	48.00	-5.12	....	2530	370
HD97334	0.60	7.60	-4.51	<2920	529	77
HD98712	1.36	11.60	-4.08	....	194	20
HD101501	0.74	16.00	-5.17	>11320	1200	170
HD114613	0.70	33.00	-5.85	5200	5600	930
HD114710	0.58	12.40	-5.50	<1120	1530	250
HD115383	0.58	3.30	-4.82	<760	120	16
HD115404	0.92	19.00	-5.25	....	1020	130
HD128620	0.71	29.00	-6.45	7840	4200	670
HD128621	0.88	42.00	-5.97	>11360	5170	820
HD131156A	0.72	6.20	-4.70	<760	207	24
HD131977	1.11	44.60	-5.38	<600	3690	560
HD141004	0.60	18.00	-6.18	6320	2780	460
HD147513	0.62	8.50	-4.61	<680	587	84
HD147584	0.55	13.00	-4.58	....	2080	370

8. Ages via gyrochronology for components of wide binaries

As we have seen, testing these (or other) stellar ages is complicated because no star apart from the Sun has an accurately determined age. However, it is possible to test the ages in a relative manner by asking whether the individual components of binary stars yield the same age. This test has been applied, with mixed results, to chromospheric ages by Soderblom et al. (1991) and Donahue (1998). We show here that gyrochronology yields substantially similar ages for both components of the three main sequence wide binary systems where *measured* rotation periods are available for the individual stars. [This latter requirement excludes otherwise interesting systems like 16 Cyg A/B (HD186408/HD186427; e.g. Cochran et al. 1997), where the rotation periods are derived quantities (Hale 1994), and 70 Oph A/B (HD165341A/B), in the Mt. Wilson sample, where only the A component has a measured period (Noyes et al. 1984; Baliunas et al. 1996).]

8.1. ξ Boo A/B (HD131156A/B)

ξ Boo A/B is a wide main sequence binary (G8V+K4V) in the Mt. Wilson sample. The orbit calculated by Hershey (1977) gives a period of 152 yr and eccentricity of 0.51, suggesting no rotational interaction between the components. The Mt. Wilson datasets (Noyes et al. 1984; Baliunas et al. 1996; Donahue et al. 1996) provide separate color and period measurements for both components, making the system particularly valuable as a test of the mass dependence of rotation, under the assumption that binarity does not affect their rotation. Since the components of binaries are usually considered to be coeval, gyrochronology ought to give the same age for the individual components. For this system, gyrochronology yields ages of 187 Myr and 265 Myr for the bluer and redder components respectively (Table 7), which gives a formal mean age for the system of 226 $\pm$18Myr. The individual values, though not in agreement within the formal errors, are closer together than those provided by other methods. For example, the chromospheric ages for the components are 232 Myr and 508 Myr respectively, which also suggest a young age for the system. As regards isochrone ages, Fernandes et al. (1998) have derived an isochrone age for the system of 2 $\pm$ 2Gyr. More recently, Takeda et al. (2007) have derived isochrone ages for the A and B components of $<$0.76 Gyr and $>$12.60 Gyr respectively, attesting to the difficulty of applying the isochrone method to field stars.

8.2. 61 Cyg A/B (HD201091/HD201092)

There is a second, lower mass, main sequence wide binary (K5V+K7V) in the Mt. Wilson sample for which measured colors and periods are available. This is the 61 Cyg A/B visual binary system, whose parameters (from Donahue et al. 1996, see also Baliunas et al. 1996 and Hale, 1994) are listed in Table 7. The orbit from Allen et al. (2000) suggests a semi-major axis of 85.6 AU and eccentricity of 0.32, while that from Gorshanov et al. (2005) suggests a period of 659yr and eccentricity of 0.48. Neither of these suggest an interaction between the components. Gyrochronology yields ages of 2.12 Gyr and 1.87 Gyr for the A and B components respectively, suggesting a mean age for the system of 2.0$\pm$0.2 Gyr (see Fig. 16), where the large differential rotation of the components contributes significantly to the error. The corresponding chromospheric ages for the same stars are 2.36 Gyr and 3.75 Gyr respectively, again in reasonable agreement, but not as close as the gyro ages. The isochrone ages for these stars, upper limits of $<$0.44 Gyr and $<$0.68 Gyr respectively (Takeda et al. 2007), seem somewhat short.

8.3. αCenA/B (HD128620/HD128621)

We now consider the famous older system αCenA/B, its G2V and K1V components bracketing the Sun in mass, a system much studied by many researchers over the years (e.g. Guenther and Demarque, 2000; Miglio & Montalban 2005), and of special interest to asteroseismologists. Heintz (1982) has calculated an orbit with period of ∼80yr and eccentricity of 0.516, suggesting that the components have not suffered rotational interactions. The published ages for the system range from 4 Gyr to 8 Gyr, depending on the details of the models used (see, e.g. Guenther & Demarque 2000). Guenther and Demarque (2000) themselves derive an age range of 7.6-6.8 Gyr, somewhat older than the Sun, depending on whether or not αCenA has a convective core. Eggenberger et al. (2004) suggest an age of 6.5±0.3 Gyr. Using the rotation periods provided by Ed Guinan (2006, personal communication), 28±3d and 36.9±1.8d for the A and B components respectively [24], and $B-V$ colors[25] of 0.67±0.02 and 0.87±0.02, we derive ages for the components of 4.6 Gyr and 4.1 Gyr, with a mean of 4.4±0.5 Gyr, toward the lower end of the published ages[26], but in good agreement with one another[27]. These stars, and the corresponding isochrone are also plotted in Fig. 16. The chromospheric ages for the αCen A and B components using R'_{HK} values from Henry et al. (1996) are 5.62 and 4.24 Gyr respectively, again comparable, if not as close. In comparison, the isochrone ages from Takeda et al. (2007) for the A and B components, derived separately, are 7.84 Gyr and >11.36 Gyr respectively.

8.4. 36 Oph A/B/C (HD155886/HD155885/HD156026)

Finally, we consider the triple system 36 Oph A/B/C, included in the Mt. Wilson sample. A and B are two chromospherically active K1 dwarfs, while the distant tertiary, C, is a K5 dwarf. The AB orbit has a period of ∼500yr, but a very high eccentricity of ∼0.9, implying a closest approach of A and B of order 6AU (Brosche 1960; Irwin et al. 1996). The latter fact suggests proceeding with caution, because A and B could potentially have interacted rotationally.

We have used the observed periods of 20.69d, 21.11d, and 18.0d, listed in Donahue et al. (1996) and Baliunas et al. (1983) for the A, B, and C components respectively to plot these in the color-period diagram displayed in Fig. 17[28]. The gyro ages for A and B are both nominally 1.43 Gyr, but that for C is only 590±70 Myr. We favor the lower age here because the C component is distant, while the A and B components seem to have interacted and presumably spun down to their ∼21d periods from the ∼13.4d periods that would otherwise be expected for the 590 Myr age for the system.

Interestingly, the chromospheric ages for the A, B, and C components range from 1.1 Gyr to 1.4 Gyr,

---

[24] Pizzolato et al. (2003) lists periods of 29d and 42d respectively, sourced from Saar and Osten (1997), which in turn sources the first to Hallam, Aliner & Endal (1991), and states that the latter is estimated from CaII measurements.

[25] SAB thanks David Frew for his trouble researching these colors.

[26] The Pizzolato et al. (2003) periods would yield a slightly older gyro age of 4.6 Gyr for the system.

[27] There is a third component in the αCen system, αCen C (Proxima Centauri), and it too has a measured period, 31±2d, but its spectral type is M5V, so it is not on the interface sequence (and hence not considered here), and it ought to follow the age dependence appropriate for the C sequence stars, but this dependence is not yet known well.

[28] Baliunas et al (1996) list a joint period of 21d for all three components. Pizzolato et al. (2003) reference Saar & Osten (1997) for the 20.69d and 21.11d periods for A and B, and Hempelmann et al. (1995), who in turn references Noyes et al. (1984) for the 18.0d (observed) period for C. Saar & Osten (1997) themselves reference Donahue et al. (1996) for the A and B periods, and say that the 18.5d period is estimated from CaII measurements.

similar to the gyro age for the A/B pair. The isochrone age for the C component only, provided by Takeda et al. (2007) is <480 Myr, again suggesting a youthful system. The fact that the A and B components have essentially the same mass provides a simplification that could be quite useful to further studies of this system.

For the present state of gyrochronology, we consider the particular cases presented above to represent success.

Table 6—Continued

Star	$B-V$	P_{rot}(d)	$log(L_x/L_{bol})$	t_{iso}/Myr	t_{gyro}/Myr	δt_{gyro}/Myr
HD149661	0.81	23.00	-4.96	<4160	1950	280
HD152391	0.75	11.10	-4.66	720	574	73
HD154417	0.58	7.60	-4.91	4200	597	91
HD155885	0.86	21.11	-4.71	....	1440	200
HD155886	0.85	20.69	-4.65	....	1420	190
HD156026	1.16	18.00	-5.23	<480	593	71
HD160346	0.96	36.00	-5.36	....	3210	480
HD165185	0.62	5.90	-4.43	....	291	39
HD165341	0.86	19.70	-5.18	....	1260	170
HD166620	0.87	42.00	-6.19	>11200	5300	845
HD176051	0.59	16.00	-5.70	....	2350	390
HD185144	0.79	29.00	-5.58	....	3220	490
HD187691	0.55	10.00	-5.97	3200	1250	210
HD190007	1.12	29.30	-5.01	<1760	1620	220
HD190406	0.61	14.50	-5.58	3160	1730	270
HD191408	0.87	45.00	-6.42	>7640	6050	980
HD194012	0.51	7.00	-5.49	....	900	170
HD201091	1.17	37.90	-5.51	<440	2450	350
HD201092	1.37	48.00	-5.32	<680	2960	440
HD206860	0.58	4.70	-4.62	<880	237	34
HD209100	1.06	22.00	-5.69	....	1030	130
HD216803	1.10	10.30	-4.54	<520	223	23
HD219834B	0.91	42.00	-5.49	>13200	4820	760
HD224930	0.67	33.00	-5.90	....	6330	1100

[A]This gyro age should be treated with caution because this star is a spectroscopic binary (see text, and Pourbaix, 2000).

9. Comparison with isochrone ages

A uniform comparison of gyro- and isochrone ages was not possible until Takeda et al. (2007) submitted a manuscript to the Astrophysical Journal Supplement subsequent to this submission. This paper contains a very careful derivation of isochrone ages for the ∼1000 stars in the Spectroscopic Properties of Cool Stars Catalog (SPOCS). This catalog (Valenti & Fischer 2005) itself consists of high-resolution echelle spectra and their detailed analysis of over 1000 nearby F-, G- and K-type stars obtained through the Keck, Lick, and Anglo-Australian Telescope planet search programs, including ∼100 stars with known planetary companions.

Takeda et al. (2007) conduct a Bayesian analysis of the stellar parameters using reasonable priors to generate a probability distribution function (PDF) for the age of each star. This method permits the identification of a 'well-defined' age for a star if the PDF peaks appropriately, or just as importantly, the derivation of an isochrone upper- or lower limit for the age. Indeed, for most of the stars common to our sample and theirs, they derive only such a limit, as a glance at the column for isochrone ages in Tables 3, 5 and 6 shows. However, for 26 of these (common) stars, Takeda et al. (2007) list well-defined ages, and these can be compared to the corresponding gyro ages.

This comparison is shown in Fig. 18. Of these 26 stars, only 3 lie above the line of equality, and 13 have isochrone ages within a factor of two of the gyro ages, all higher than the corresponding gyro ages. In fact, the median isochrone age is 2.7 times the median gyro age. Evidently, the Bayesian technique used still does not eliminate the known bias in the isochrone ages towards older values.

In fact, the same test applied to the binary systems in the previous section with respect to gyro ages yields uncertain results with respect to these isochrone ages. Indeed, of the 9 stars under consideration, only one (α Cen A) has a well-defined isochrone age, and the rest upper- or lower limits. These stars are also plotted in Fig. 18, with dashed lines joining the binary components, and arrows indicating upper- or lower limits.

In summary, it would seem that the isochrone ages are still problematical, despite the careful analysis of Takeda et al. (2007). Of course, as we have noted in the introduction, it is perhaps not fair and evidently not possible, to use slowly varying parameters to derive precise ages for stars on the main sequence. The two methods are, however, complementary in that it might be preferable to use gyro ages on the main sequence, and isochrone ages off it.

10. Conclusions

The rotation period distributions of solar and late-type stars suggest that coeval stars are preferentially located on one of two sequences. The mass- and age dependencies of one of these sequences, the interface sequence, are shown to be universal, shared by both cluster and field stars, and we have specified them using simple functions, generalizing the dependence originally suggested by Skumanich (1972). The mass dependence is derived observationally using a series of open clusters, and the age dependence, roughly $\sqrt{t}$, is specified via a solar calibration.

The dependencies are inverted to provide the age of a star as a function of its rotation period and color, a procedure we call gyrochronology. Errors are calculated for such ages, based on the data currently available, and shown to be roughly 15% (plus possible systematic errors) for individual stars. Because the dependencies are universal, they must also apply to field stars, but the derivation of such ages requires excising pre-I sequence stars, facilitated by their location below the I sequence in color-period diagrams. The

short lifetime of this pre-I sequence phase assures us that all such stars are less than a couple of hundred million years in age.

Using this formalism, we have calculated ages via gyrochronology for individual stars in three illustrative groups of field stars, and listed them along with the errors. For the first group, the Mt. Wilson stars, these ages are shown to be in general agreement with the chromospheric ages, except that stars bluer than the Sun have systematically higher chromospheric ages, the median chromospheric age being higher by about 33%. The majority of the second group, from Strassmeier et al. (2000), are shown to be younger than 1 Gyr, in keeping with the selection of the sample for activity, which correlates negatively, as expected, with gyro age. The third group, from Pizzolato et al. (2003), are shown to be somewhat older, partially due to an overlap with the Mt. Wilson sample, and their X-ray fluxes are shown to decay systematically with gyro age. We have shown that gyrochronology yields similar ages for both components of three wide binary systems, ξ Boo A/B, 61 Cyg A/B, and α Cen A/B. The 36 Oph A/B/C triple system shows signs of rotational interaction between the A and B components. Finally, the recent Takeda et al. (2007) isochrone ages appear to be inferior to the gyro ages for the same main sequence stars.

Thus, we have re-investigated the use of a rotating star as a clock, clarified and improved its usage, calibrated it using the Sun, and demonstrated that it keeps time well.

The word “gyrochronology” was inspired by the work of A. E. Douglass on dendrochronology at Lowell Observatory. SAB would like to acknowledge Sabatino Sofia as a constant source of intellectual and moral support and many discussions, and Charles Bailyn for initially suggesting the removal of the age dependence. Marc Buie, Will Grundy, Wes Lockwood, Bob Millis, Byron Smith, Brian Skiff and my other colleagues at Lowell have supported me in numerous ways. Stephen Levine read the manuscript closely, and found an algebraic error. David James, Heather Morrison, Steve Saar, Sukyoung Yi and an anomymous referee are gratefully acknowledged for input on a prior version of the paper. The paper owes much to the baristas at Late For The Train, Flagstaff. Finally, this material is based upon work partially supported by the National Science Foundation under Grant No. AST-0520925.

A. Appendix: Derivation of the error on the index n

By definition,

$$P = f(B-V).g(t) = a\,x^b\,t^n \tag{A1}$$

Taking natural logarithms and rearranging, we get

$$n = \frac{ln\,P_\odot - ln\,a - b\,ln\,x_\odot}{ln\,t_\odot} = \frac{U}{V} \tag{A2}$$

Differentiating yields

$$\frac{dn}{n} = \frac{dU}{U} - \frac{dV}{V} \tag{A3}$$

or

$$\frac{dn}{n} = \frac{1}{U}[\frac{dP_\odot}{P_\odot} - \frac{da}{a} - b\frac{dx_\odot}{x_\odot} - ln\,x_\odot\,db] - \frac{dt_\odot}{t_\odot\,ln\,t_\odot} \tag{A4}$$

Adding the errors in quadrature yields

$$(\frac{\delta n}{n})^2 = (\frac{\delta t_\odot}{t_\odot\,ln\,t_\odot})^2 + \frac{1}{U^2}[(\frac{\delta P_\odot}{P_\odot})^2 + (\frac{\delta a}{a})^2 + (b\frac{\delta x_\odot}{x_\odot})^2 + (ln\,x_\odot\,\delta b)^2] \tag{A5}$$

For the error in the age of the Sun (4566 Myr; $ln\, t_\odot = 8.426$), we adopt the value of 50 Myr[29], for that in the rotation period, 1 d (consistent with the measured range in the solar rotation period - see section 4 and Donahue et al. 1996), and for that in the solar $B-V$ color ($x = B - V_\odot = 0.242$), we adopt the value 0.01. From section 2, $a = 0.7725 \pm 0.011$ and $b = 0.601 \pm 0.024$. Input of these values yields

$$(\frac{\delta n}{n})^2 = (\frac{50}{4566 \times 8.43})^2 + \frac{1}{4.37^2}[(\frac{1}{26.09})^2 + (\frac{0.011}{0.7725})^2 + (0.601\frac{0.01}{0.242})^2 + (-1.419 \times 0.024)^2] \tag{A6}$$

or

$$(\frac{\delta n}{n})^2 = 1.69 \times 10^{-6} + 10^{-6}[77.4 + 10.6 + 32.3 + 60.7] = 182.6 \times 10^{-6} \tag{A7}$$

or[30],

$$\frac{\delta n}{n} = 1.37 \times 10^{-2} \tag{A8}$$

so that

$$n = 0.5189 \pm 0.0070 \tag{A9}$$

which shows that the index n is determined well.

REFERENCES

Allain, S., Bouvier, J., Prosser, C., Marschall, L.A. and Laaksonen, B.D., 1996, A&A, 305, 498

Allegre, C.J., Manhes, G., & Gopel, C., 1995, Geochemica et Cosmochimica Acta, 59(8), 1445

Allen, C., Poveda, A, & Herrera, M.A., 2000, A&A, 356, 529

Baliunas, S., Vaughan, A.H., Hartmann, L., Middelkoop, F., Mihalas, D., Noyes, R.W., Preston, G.W., Frazer, J., & Lanning, H., 1983, ApJ, 275, 752

Baliunas, S., & 27 co-authors, 1995, ApJ, 438, 269

Baliunas, S., Sokoloff, D. & Soon, W., 1996, ApJL, 457, 99

Barnes, S.A., 1998, PhD Thesis, Yale University

Barnes, S.A., 2001, ApJ, 561, 1095

Barnes, S.A., 2003a, ApJ, 586, 464

Barnes, S.A., 2003b, ApJL, 586, 145

Barnes, S.A. & Sofia, S., 1996, ApJ, 462, 746

[29] Allegre et al. (1995) list the impressively small error of $+2/-1$ Myr (in agreement with the present day precision of radioactive dating techniques) for the age of the formation of the Allende refractory inclusions, generally accepted as the age of the Earth/meteorites/Solar System. However, we astronomers do not know what event in the Sun's history corresponds to this point. Is this the zero age main sequence, or the birthline 43 Myr earlier (Barnes & Sofia, 1996), or some other event entirely? In view of these uncertainties, we adopt an error of 50 Myr in the age of the Sun.

[30] Note that the largest terms come from the differential rotation of the sun and the index b, while the age error of the sun contributes little to the error in n.

Barnes, S.A. & Sofia, S., 1998, BAAS, 30, 917

Barnes, S.A., Sofia, S., Prosser, C.F. & Stauffer, J.R., 1999, ApJ, 516, 263

Basri, G., Marcy, G.W., & Graham, J., 1996, ApJ, 458, 600

Brosche, P., 1960, AN, 285, 261

Cochran, W.D., Hatzes, A.P., Butler, R.P. , & Marcy, G.W., 1997, ApJ, 483, 457

Cowley, A.P., 1976, PASP, 88, 95

Demarque, P.D. & Larson, R.B., 1964, AJ, 140, 544

Demarque, P., Woo, J.-H., Kim, Y.-C., Yi, S.K., 2004, ApJS, 155, 667

Donahue, R.A., 1998, in ASP Conf. Ser. 154, Tenth Cambridge Workshop on Cool Stars, Stellar Systems and the Sun, ed. R.A. Donahue and J.A. Bookbinder, 1235

Donahue, R.A., Saar, S.H. & Baliunas, S.L., 1996, ApJ, 466, 384

Dorren, J.D., & Guinan, E.F., 1994, ApJ, 428, 805

Duncan, D.K., 1984, AJ, 89, 515

Duncan, D.K., Baliunas, S.L., Noyes, R.W., Vaughan, A.H., Frazer, J., & Lanning, H.H., 1984, PASP, 96, 707

ESA, 1997, The Hipparcos and Tycho Catalogues, ESA SP-1200

Eggenberger, P., Charbonnel, C., Talon, S., Meynet, G., Maeder, A., Carrier, F., & Bourban, G., 2004, A&A, 422, 247

Fernandes, J., Lebreton, Y., Baglin, A. & Morel, P., 1998, A&A, 338, 455

Giampapa, M.S., Hall, J.C., Radick, R.R., & Baliunas, S.L., 2006, ApJ, 651, 444

Girardi, L., Bertelli, G., Bressan, A., Chiosi, C., Groenewegen, M.A.T., Marigo, P., Salasnich, B., Weiss, A., 2002, A&A, 391, 195

Gorshanov, D.L., Shakht, N.A., Kisselev, A.A., & Poliakow, E.V., 2005, in *Dynamics of Populations of Planetary Systems*, eds. Z. Knezevic & A. Milani, IAU Coll 197, p.91

Gudel, M., 2004, A&ARv, 12, 71

Guenther, D.B., & Demarque, P., 2000, ApJ, 531, 503

Hale, A., 1994, AJ, 107, 306

Hallam, K.L., Aliner, B., & Endal, A.S., 1991, ApJ, 372, 610 111, 321

Heintz, W.D., 1982, Observatory, 102, 42

Hempelmann, A., Schmitt, J.H.M.M., Schultz, M., Rudiger, G., & Stepien, K., 1995, A&A, 294, 515

Henry, G.W., Fekel, F.C. & Hall, D.S., 1995, AJ, 110, 2926

Henry, T.J., Soderblom, D.R., Donahue, R.A. , & Baliunas, S.L., 1996, AJ, 111, 439

Hershey, J.L., 1977, AJ, 82, 179

Holmberg, J., Flynn, C. & Portinari, L., 2006, MNRAS, 367, 449

Ihaka & Gentleman, 1996, Journal of Computational and Graphical Statistics, 5, 299

Irwin, A.W., Yang, S.L.S., & Walker, G.A.H., 1996, PASP, 108, 580

James, D.J.J., Barnes, S.A., & Meibom, S., 2007, ApJ, submitted.

Jorgensen, B.R. & Lindegren, L., 2005, A&A, 436, 127

Kawaler, S.D., 1989, ApJ, 343, 65

Kim, Y.-C., Demarque, P., Yi, S.K., Alexander, D.R., 2002, ApJS, 143, 499

Kraft, R., 1967, ApJ, 150, 551

Krishnamurthi, A. & 15 co-authors, 1998, ApJ, 493, 914

Kunte, P.K., Rao, A.R., Vahia, M.N., 1988, Ap&SS, 143, 207

Lachaume, R., Dominik, C., Lanz, T., & Habing, H. J., 1999, A&A, 348, 897

Latham, D.W., Stefanik, R.P., Torres, G., Davis, R.J., Mazeh, T., Carney, B.W., Laird, J.B. & Morse, J.A., 2002, AJ, 124, 1144

Matthews, J.M., Kuschnig, R., Walker, G.A., Pazder, J., Johnson, R., Skaret, K., Shkolnik, E., Lanting, T., Morgan, J.P., & Sidhu, S., 2000, in The Impact of Large-Scale Surveys on Pulsating Star Research, ASP Conf. Series, 203, 74

Miglio, A. & Montalban, J., 2005, A&A, 441, 615

Murthy, V.R. & Patterson, C.C., 1962, J. Geophys. Res., 67, 1161

Noyes, R.W., Hartmann, L.W., Baliunas, S.L., Duncan, D.K. & Vaughan, A.H., 1984, ApJ, 279, 763

Patten, B.M. & Simon, T., 1996, ApJS, 106, 489

Patterson, C.C., 1953, in Report by the Subcommittee on Nuclear Processes in Geological Settings, National Research Council, National Academy of Sciences, Washington, D.C., p.36

Patterson, C.C., 1955, Geochimica et Cosmochimica Acta, 7, 151

Patterson, C.C., 1956, Geochimica et Cosmochimica Acta, 10, 230

Patterson, C.C., Tilton, G.R., & Ingraham, M.G., 1955, Science, 121, 69

Pallavicini, R., Golub, L., Rosner, R., Vaiana, G.S., Ayres, T., & Linsky, J.L., 1981, ApJ, 248, 279

Perryman, M.A.C, Lindegren, L., Kovalevsky, J., Hog, E., & 16 co-authors, A&A, 323, L49

Pinsonneault, M. H., Kawaler, S. D., Sofia, S. and Demarque, P., 1989, ApJ, 338, 424

Pizzolato, N., Maggio, A., Micela, G., Sciortino, S., & Ventura, P., 2003, A&A, 397, 147

Pojmanski, G., in Small Telescope Astronomy on Global Scales, eds. B. Paczynski, W.-P. Chen, & C. Lemme, ASP Conf. Series, 246, 53

Pont, F. & Eyer, L., 2004, MNRAS, 351, 487

Pourbaix, D., 2000, A&AS, 145, 215

Prosser, C.F. & Grankin, K., 1997, CfA preprint 4539

Radick, R.R., Thompson, D.T., Lockwood, G.W., Duncan, D.K, and Baggett, W.E., 1987, ApJ, 321, 459

Radick, R.R., Skiff, B.A. & Lockwood, G.W., 1990, ApJ, 353, 524

Randall, S.K., Matthews, J.M., Fontaine, G., Rowe, J., Kuschnig, R., Green, E. M., & 8 co-authors, 2005, ApJ, 633, 460

Rebolo, R., Martin, E.L., & Maguzzu, A., 1992, ApJL, 389, 83

Rucinski, S.M., & 10 co-authors, 2004, PASP, 116, 1093.

Saar, S.H., & Osten, R.A., 1997, MNRAS, 284, 803

Saio, H., and 12 co-authors, 2007, ApJ, 654, 544

Sandage, A., 1962, ApJ, 135, 349

Sandage, A., Lubin, L.M., & VandenBerg, D.A., 2003, PASP, 115, 1187

Skumanich, A., 1972, ApJ, 171, 565

Soderblom, D.R., 1985, AJ, 90, 2103

Soderblom, D.R. & Clements, S.D., 1987, AJ, 93, 920

Soderblom, D.R., Duncan, D.K., & Johnson, D.R.H., 1991, ApJ, 375, 722

Stauffer, J., 2000, in ASP Conf. Ser., 198, Stellar Clusters and Associations: Convection, Rotation, and Dynamos, ed. R. Pallavicini, G. Micela & S. Sciortino (San Francisco: ASP), 255

Strassmeier, K.G., 2006, Ap&SS, 304, 397

Strassmeier, K.G., Granzer, T., Boyd, L.J., Epand, D.H., 2000, Proc. SPIE, eds R.I. Kibrick & A. Wallander, 4011, 157

Strassmeier, K.G., Washuettl, A., Granzer, Th., Scheck, M., & Weber, M., 2000, A&AS, 142, 275

Takeda, G., Ford, E.B., Sills, A., Rasio, F.A., Fischer, D.A. & Valenti, J.A., 2007, ApJS, 168, 297

Valenti, J.A., & Fischer, D.A., 2005, ApJS, 159, 141

Van Leeuwen, F., Alphenaar, P., & Meys, J.J.M., 1987, A&AS, 67, 483

VandenBerg, D.A. & Stetson, P.B., 2004, PASP, 116, 997

VandenBerg, D.A., Bergbusch, P.A., & Dowler, P.D., 2006, ApJS, 162, 375

Vauclair, S. 1972, in Ages des Etoiles, Proc. IAU Coll. 17, ed. G. Cayrel de Strobel & A. M. Delplace, 38

Vaughan, A.H., 1980, PASP, 92, 392

Wilson, O.C., 1963, ApJ, 138, 832

Wozniak, P.R., Vestrand, W.T., & 16 co-authors, 2004, AJ, 127, 2436

Wright, J.T., Marcy, G.W., Butler, R.P., Vogt, S.S., 2004, ApJS, 152, 261

Yi, S.K., Demarque, P., Kim, Y.-C., Lee, Y.-W., Ree, C.H., Lejeune, T., & Barnes, S.A., 2001, ApJS, 136, 417

Table 7. Ages for wide binary systems.

Star	$B-V$	$\bar{P}_{rot}$(d)[a]	Age_{chromo}	Age_{iso}	Age_{gyro}[b]
HD131156A	0.76	6.31(0.05)	232 Myr	<760 Myr	187±21 Myr
HD131156B	1.17	11.94(0.22)	508 Myr	>12600 Myr	265±28 Myr
Mean					**226±18 Myr**
HD201091	1.18	35.37(1.3)	2.36 Gyr	<0.44 Gyr	2.12±0.3 Gyr
HD201092	1.37	37.84(1.1)	3.75 Gyr	<0.68 Gyr	1.87±0.3 Gyr
Mean					**2.0±0.2 Gyr**
HD128620	0.67	28(3)	5.62 Gyr	7.84 Gyr	4.6±0.8 Gyr
HD128621	0.87	36.9(1.8)	4.24 Gyr	>11.36 Gyr	4.1±0.7 Gyr
Mean					**4.4±0.5 Gyr**
HD155886	0.85	20.69(0.4)	1.1 Gyr	......	1.42±0.19 Gyr
HD155885	0.86	21.11(0.4)	1.2 Gyr	......	1.44±0.20 Gyr
HD156026	1.16	18.0(1.0)	1.4 Gyr	<0.48 Gyr	**0.59±0.07 Gyr**

[a]Differential rotation is the main contributor to the period errors in the parentheses.

[b]Boldface figures denote the final gyro age for each system

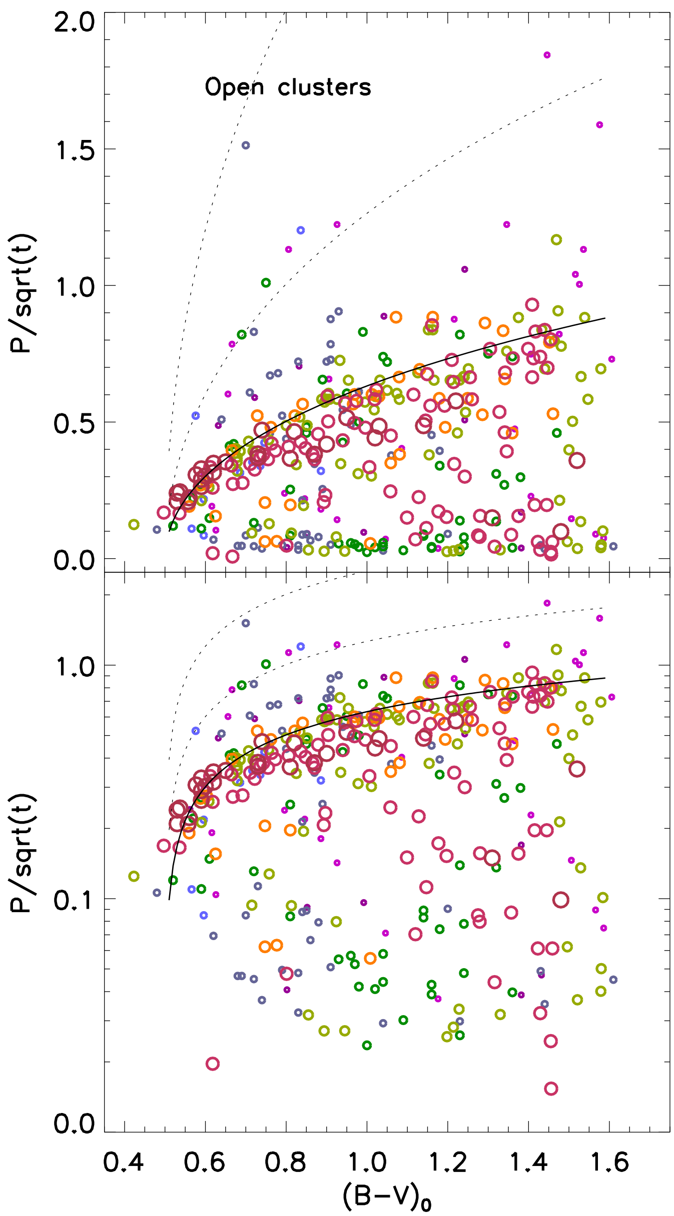

Fig. 1.— Plot of $P/\sqrt{t}$ vs $(B-V)_0$ for open cluster stars only (P=rotation period; t=cluster age). Symbol sizes and colors correspond to cluster age (blue/small = young, red/large = old). The densest concentration of stars in the vicinity of the solid line represents the interface sequence. Note how the interface sequences of all the open clusters coincide. Also note the clearly visible convective sequence along the lower edge of the upper panel. The solid line represents $f(B-V)$. Dotted lines are at $2f$ and $4f$. Some stars in the vicinity of the dashed lines could be spurious periods or non-members. The same data are plotted in both panels, on a linear scale in the upper panel, and on a logarithmic scale in the lower panel.

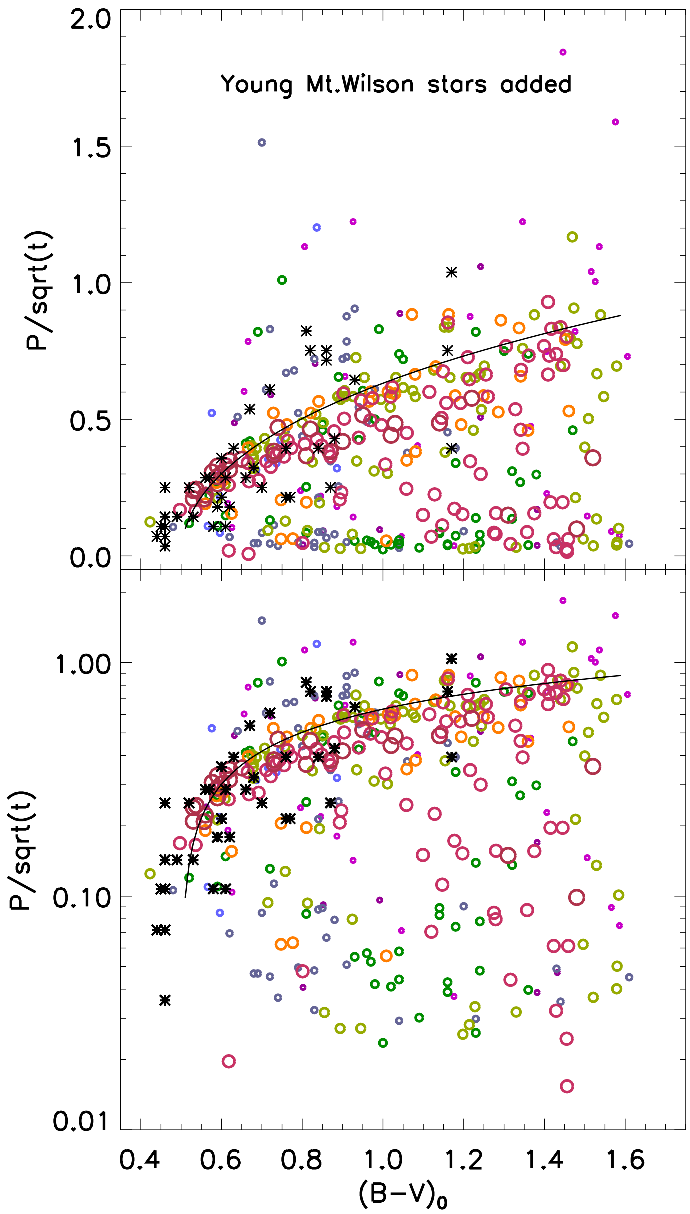

Fig. 2.— Plot of $P/\sqrt{t}$ vs $(B-V)_0$ for the young Mt. Wilson stars (small black asterisks), assumed to be 780 Myr old, the median chromospheric age for this sample, overplotted on the open cluster data. Note how the young Mt. Wilson stars overlie the interface sequences for the open clusters, and that no young Mt. Wilson stars are on the C sequence. The non-coeval nature of the young Mt. Wilson sample probably accounts for much of the dispersion observed.

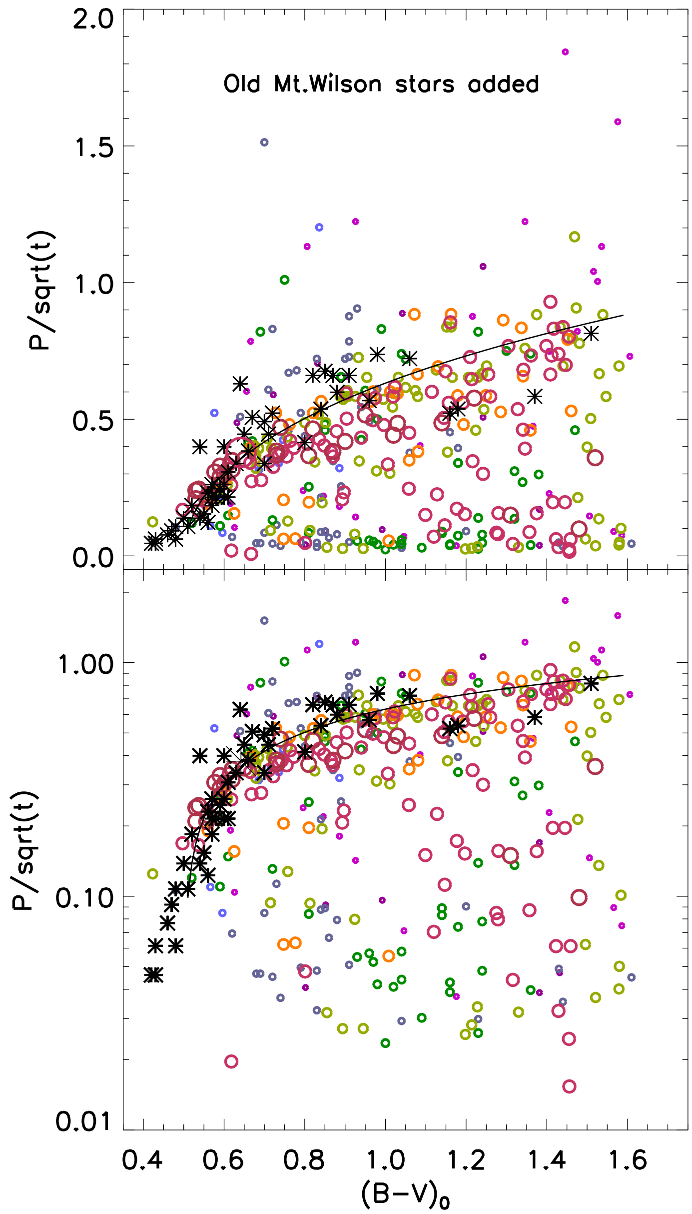

Fig. 3.— Plot of $P/\sqrt{t}$ vs $(B-V)_0$ for the old Mt. Wilson stars (large black asterisks), assumed to be 4.24 Gyr old, the median chromospheric age for this sample, overplotted on the open cluster data. Note how the old Mt. Wilson stars overlie the interface sequences for the open clusters, and that no Mt. Wilson stars are located near the C sequence. The non-coeval nature of the old Mt. Wilson sample probably accounts for much of the dispersion observed.

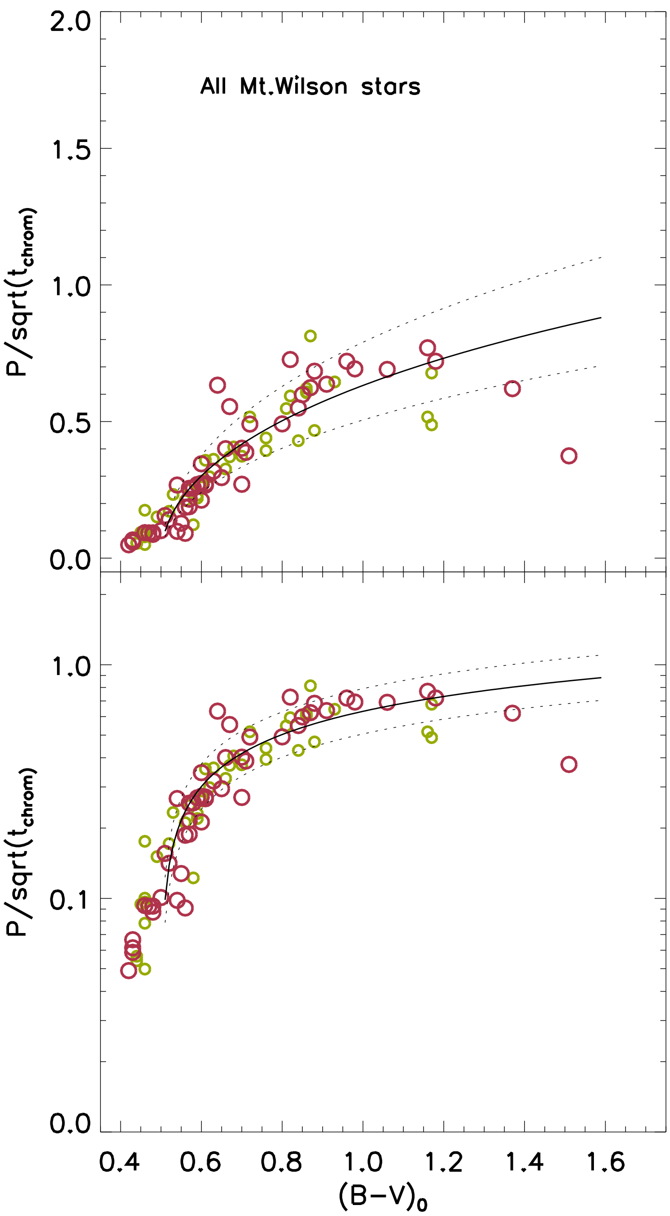

Fig. 4.— Plot of $P/\sqrt{t}$ vs $(B-V)_0$ for the individually age-corrected (chromospheric ages) Mt. Wilson stars. Note how the Mt. Wilson stars (small green circles = young; large red circles =old) lie on top of the interface sequences for the open clusters. The solid line represents $f(B-V)$, as before, and the dotted lines are a factor of $0.8-$ and $1.25 \times f$ ($\pm 25\%$).

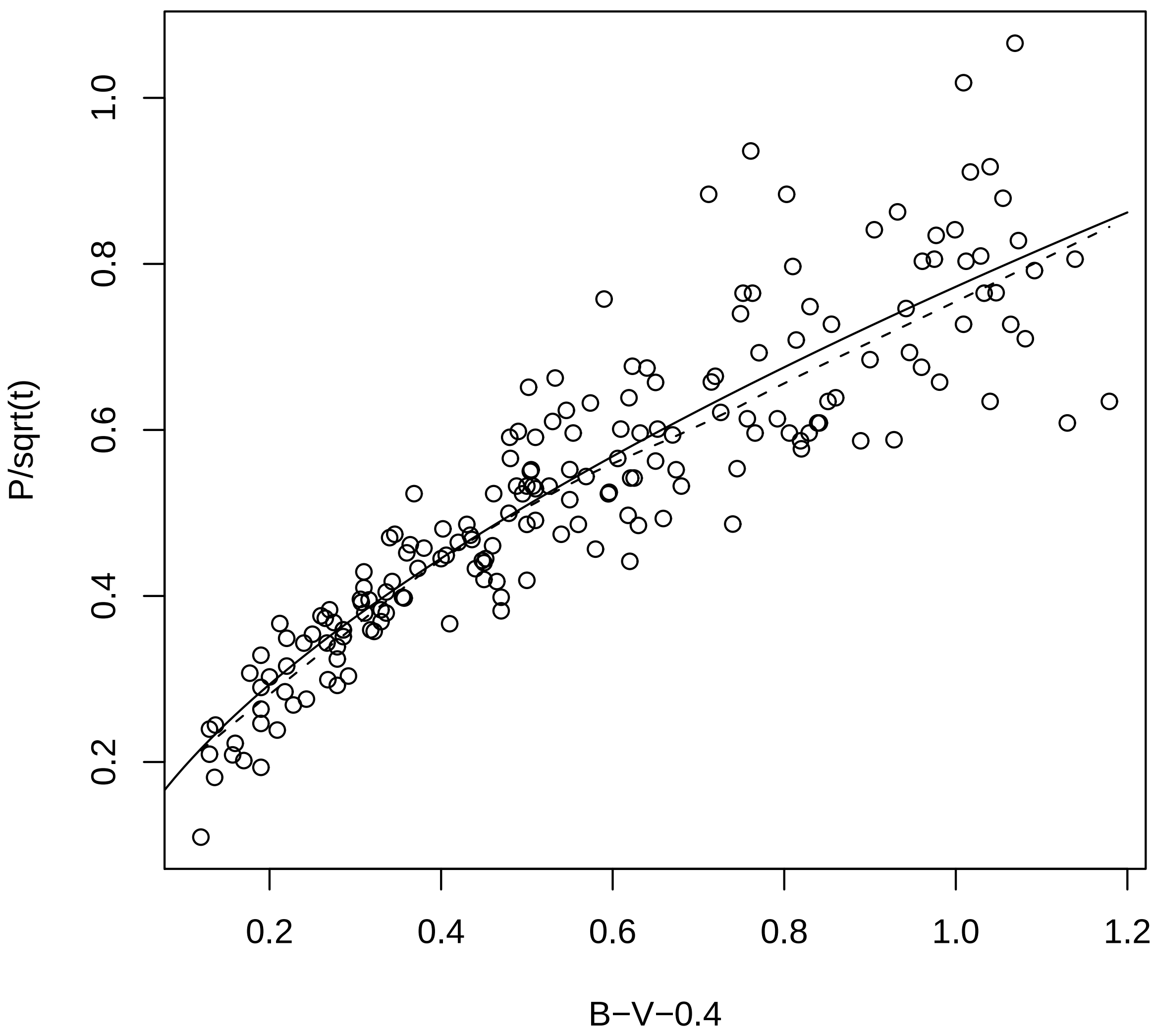

Fig. 5.— The fit to the mass dependence (solid line), using R: $f(B-V) = (0.7725 \pm 0.011) \times (B - V_0 - 0.4)^{0.601\pm0.024}$. The abscissa gives $(B - V_o - 0.4)$ and the ordinate $P/\sqrt{t}$ for individual I sequence stars in the main sequence open clusters listed in the text. The dashed line shows a smooth trend curve plotted using the function *lowess* in the R statistics package. Note the similarity of the two curves, which demonstrates that the fitting function is appropriate for these data.

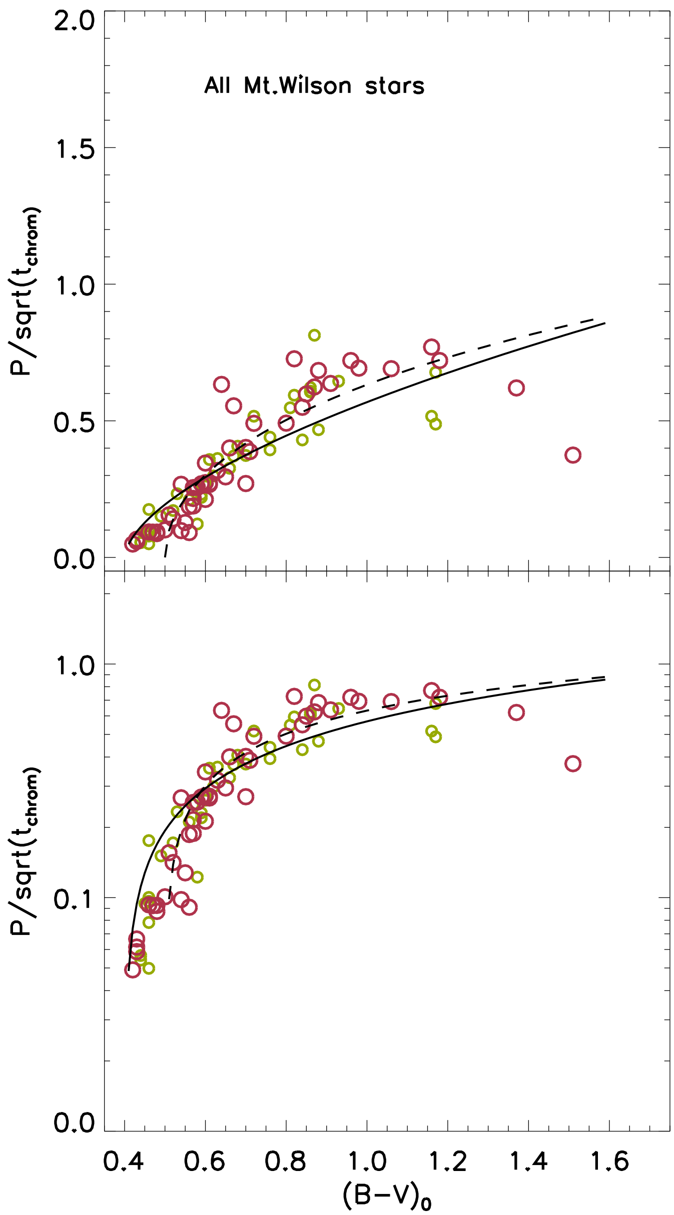

Fig. 6.— Plot of $P/\sqrt(t)$ vs $(B-V)_0$ for individually age-corrected (chromospheric ages) Mt. Wilson stars (small green circles = young; large red circles = old), with the old (dashed) and new (solid) functions, f, overplotted. Note that the new function accommodates bluer stars. The discrepancy arises from the assumed chromospheric ages for the stars, which are almost certainly overestimated for the F stars (see text). The same data are plotted in both panels, on a linear scale in the upper panel, and on a logarithmic scale in the lower panel.

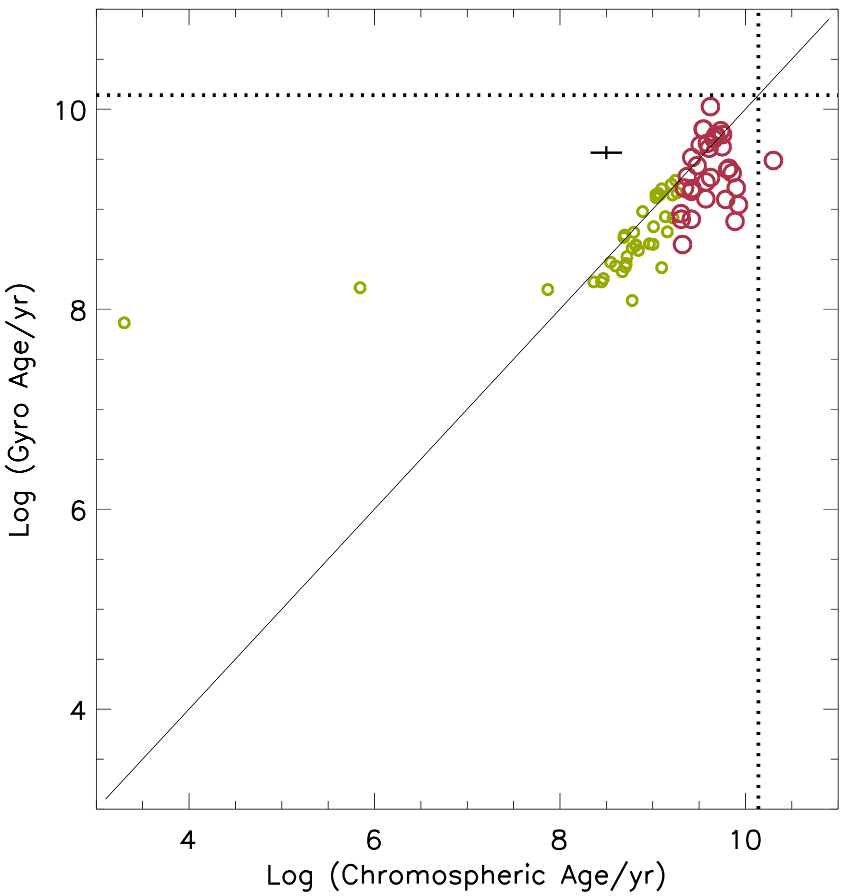

Fig. 7.— Comparison of gyro- and chromospheric ages for the Mt. Wilson stars. The young (Y) and old (O) Mt. Wilson stars are marked with small green and large red circles respectively. The line indicates equality. Note that the gyro ages are well-behaved for the youngest stars, where the chromospheric ages are suspect. The dotted lines represent the age of the universe, and the cross indicates typical gyro/chromospheric age errors quoted for this sample.

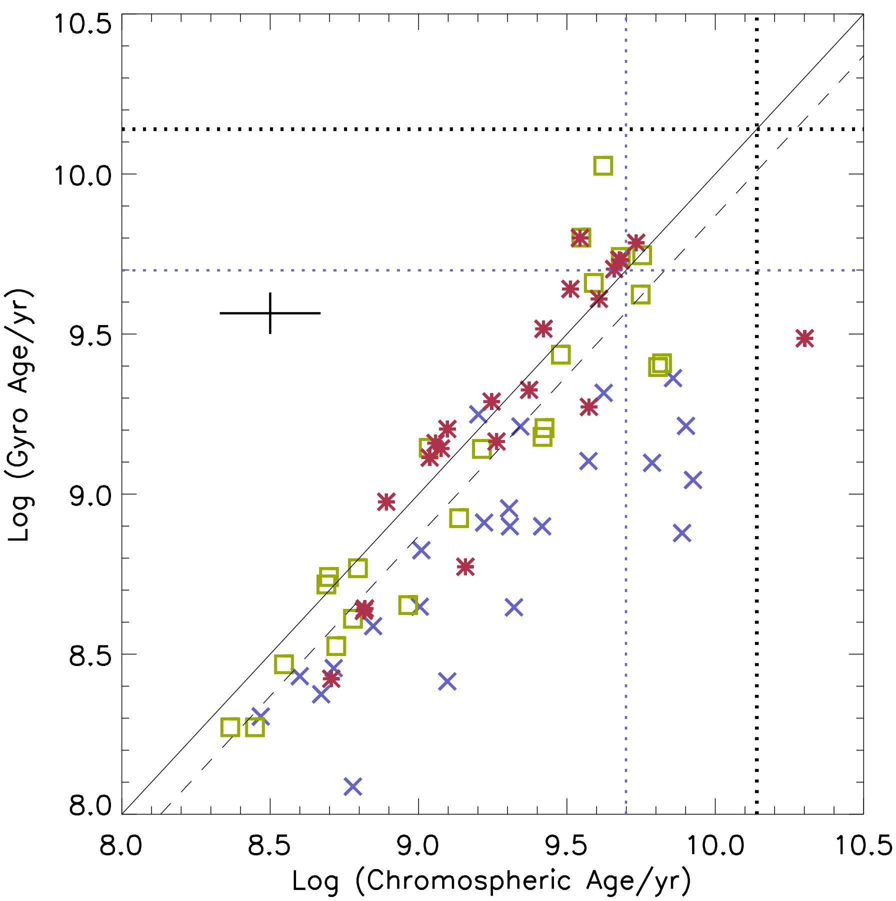

Fig. 8.— Comparison of gyro and chromospheric ages for the Mt. Wilson stars. Blue crosses indicate stars bluer than $B - V = 0.6$, red asterisks stars redder than $B - V = 0.8$, and green squares those with colors between. The upper (solid) line indicates equality, while the lower (dashed) line at $Age_{gyro} = 0.74 \times Age_{chromo}$ bisects the data. Note that both techniques are in general agreement about the youth or antiquity of any particular star, but that the gyro ages are roughly 25% lower on average. The figure also shows that the bluer stars contribute most to this discrepancy. The thick and thin dotted lines represent the age of the universe and the lifetime of F stars (5 Gyr) respectively. The cross indicates typical gyro/chromospheric age errors quoted for this sample.

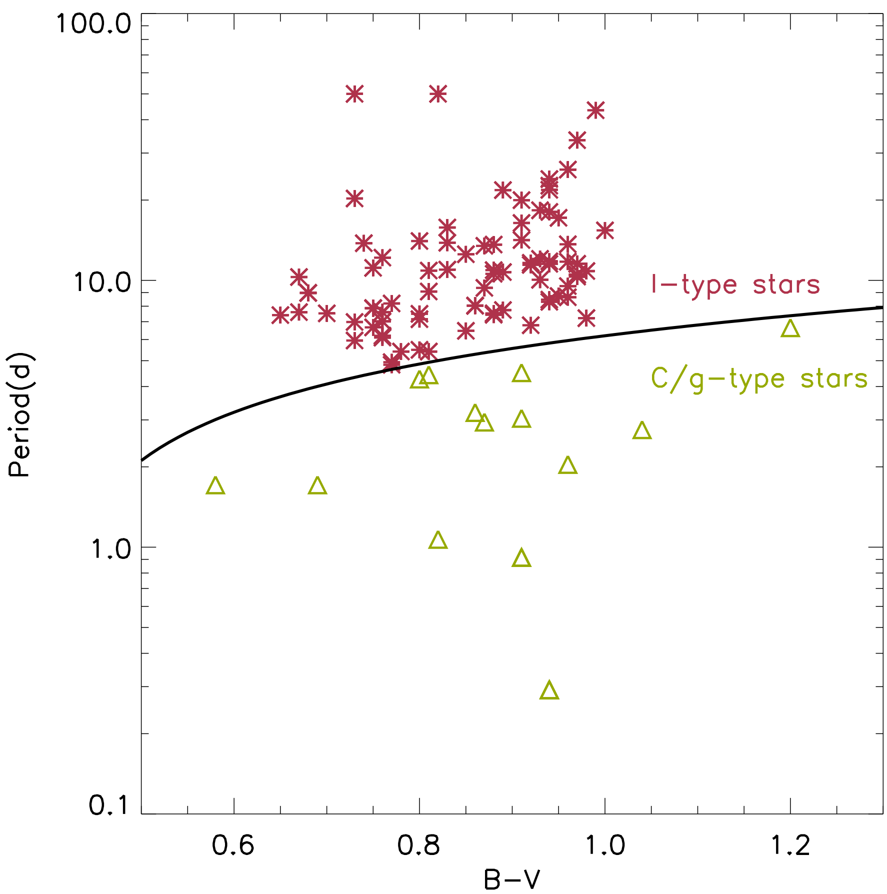

Fig. 9.— Division of the Strassmeier et al. (2000) sample into I sequence (suitable for gyrochronology) and C/g (unsuitable) categories. The solid line separates the two categories of stars, and represents an isochrone for 100 Myr.

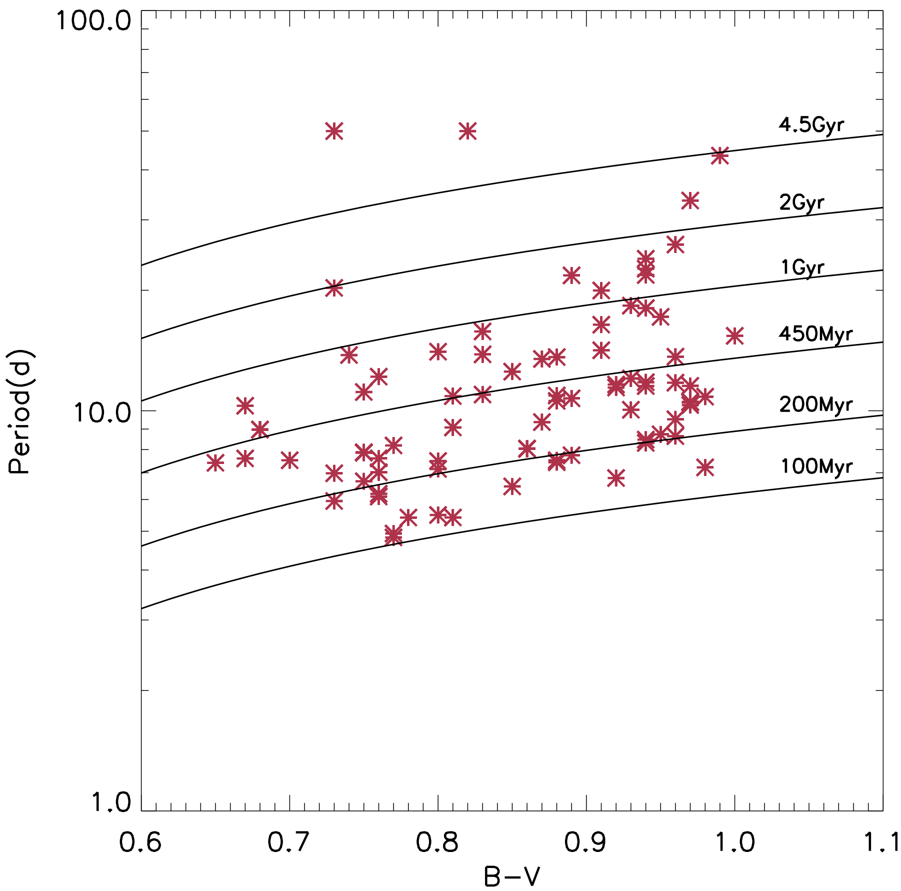

Fig. 10.— Ages for the Strassmeier et al. (2000) I sequence stars may be read off this figure. Isochrones correspond to ages of 100 Myr, 200 Myr, 450 Myr, 1 Gyr, 2 Gyr, and 4.5 Gyr. Note that all but 4 of the stars are less than 2 Gyr in age.

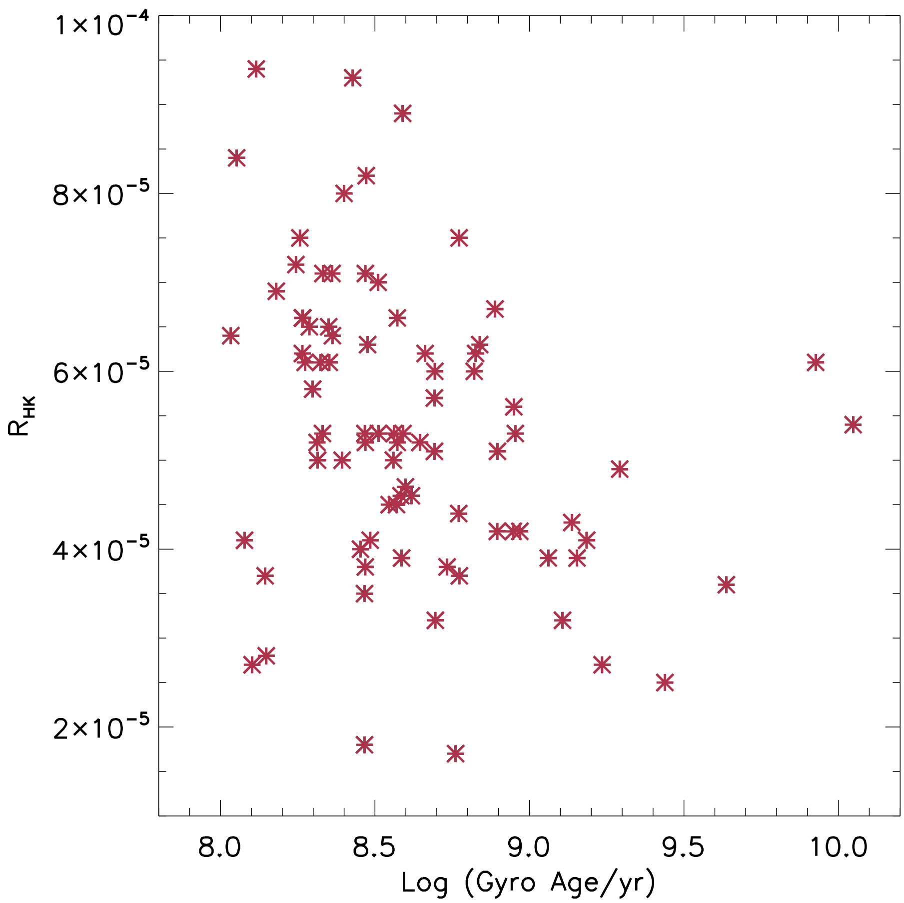

Fig. 11.— R_{HK} vs. Gyro age for the Strassmeier et al. (2000) I sequence stars. Note the declining trend of R_{HK} with age. The trend is obvious despite the fact that the R_{HK} values are not long-term averages, and have not been corrected for photospheric contributions or variation with color.

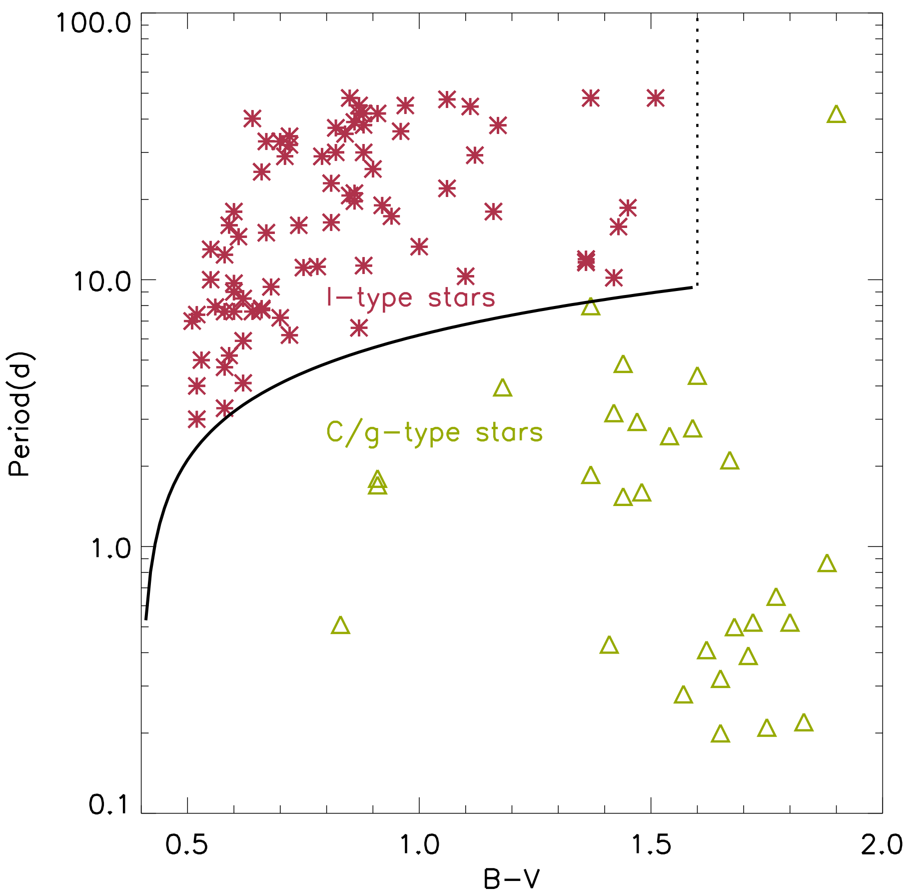

Fig. 12.— Division of the Pizzolato et al. (2003) stars into C/g- & I categories. The solid line separates the two categories of stars, and represents a rotational isochrone for 100 Myr. The dotted line indicates the approximate color ($B - V = 1.6$; M 3) for the onset of full convection.

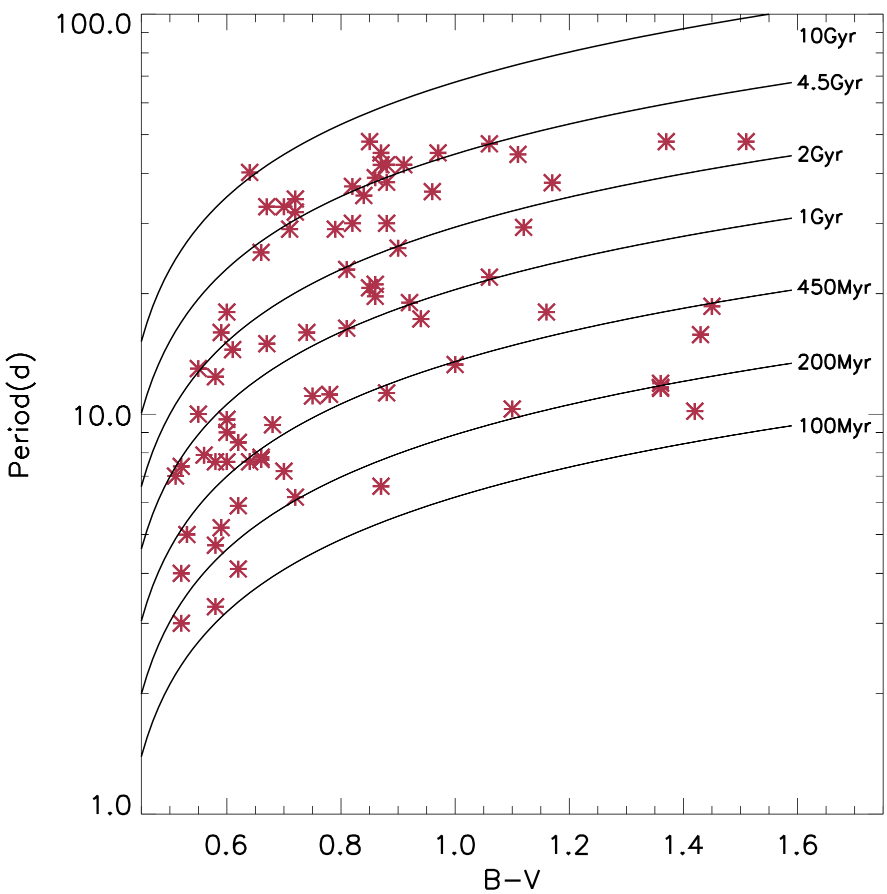

Fig. 13.— Ages for the Pizzolato et al. (2003) stars may be read off this figure. Rotational isochrones correspond to ages of 100 Myr, 200 Myr, 450 Myr, 1 Gyr, 2 Gyr, 4.5 Gyr, & 10 Gyr, as marked.

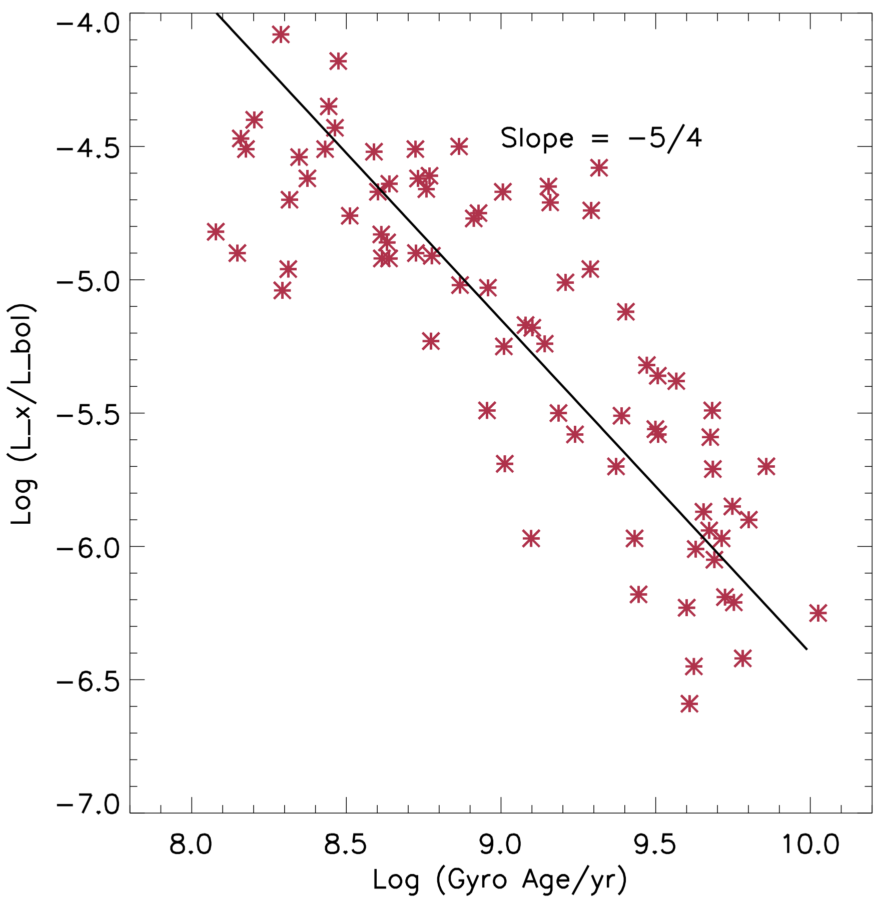

Fig. 14.— X-ray emission vs. gyro age for the Pizzolato et al. (2003) stars. Note the steady decline in X-ray emission with gyro age, as expected. The line drawn has a slope of $-5/4$, as expected from MHD turbulence.

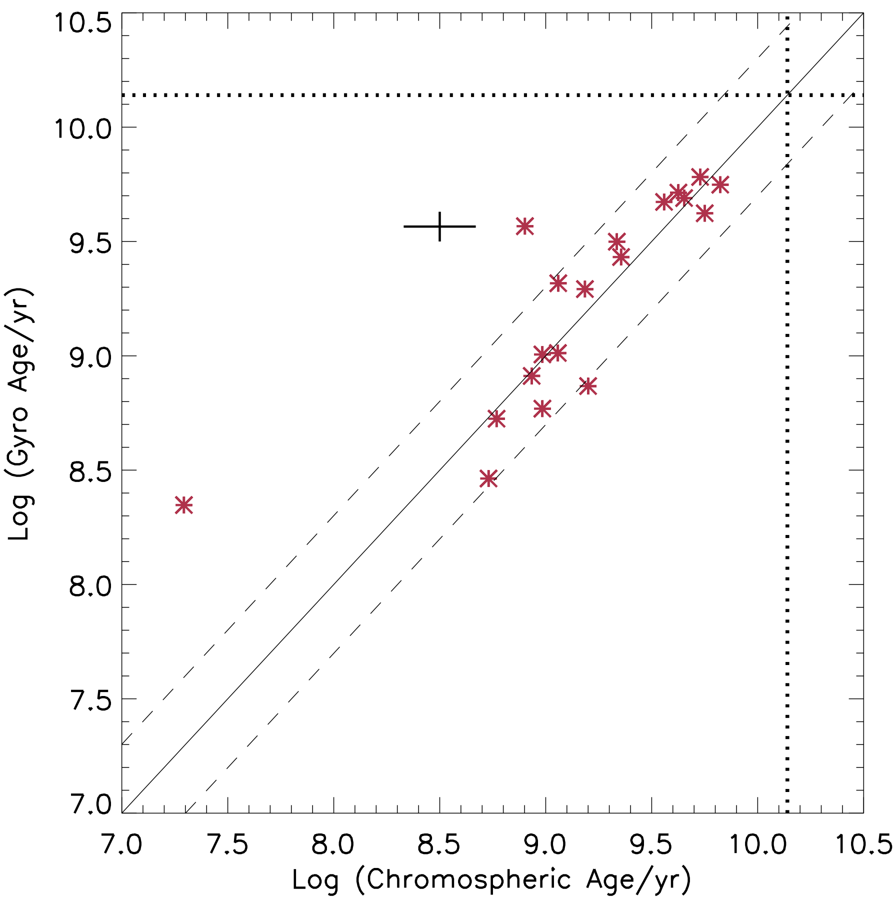

Fig. 15.— Comparison of gyro and chromospheric ages for the 19 Pizzolato et al. (2003) stars in the Southern Chromospheric Survey of Henry et al. (1996). Note that almost all the stars scatter about the line of equality (solid). The dashed lines indicate factors of two above and below the gyro ages. Typical error bars are indicated. The dotted lines indicate the age of the universe.

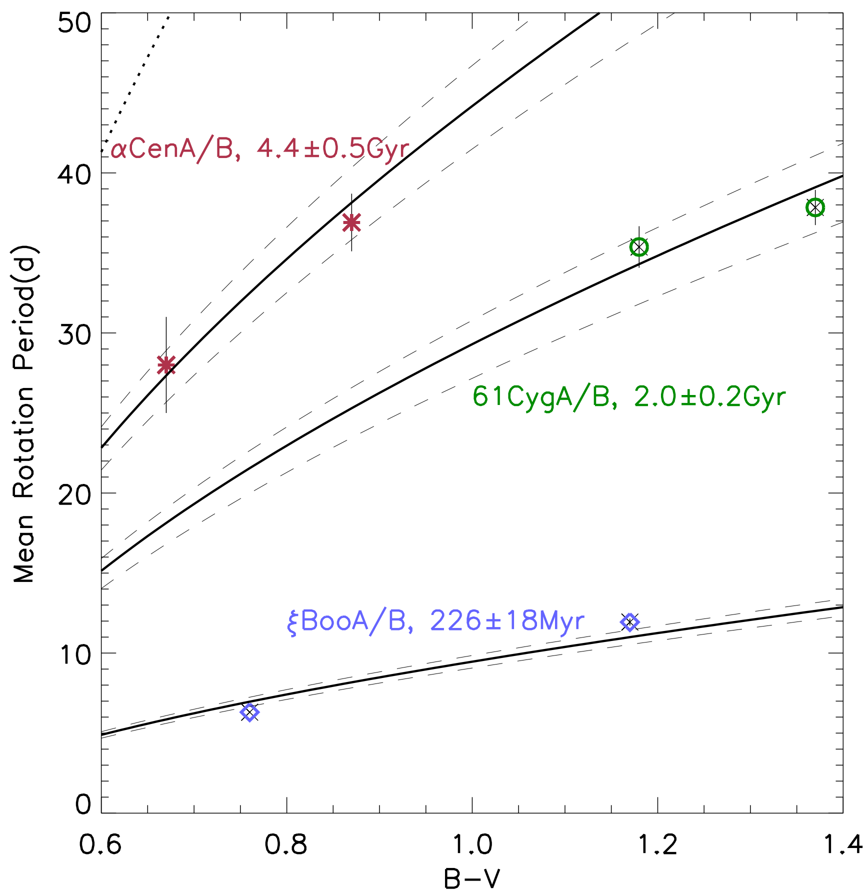

Fig. 16.— Color-period diagram for three wide binary systems, ξ Boo A/B, 61 Cyg A/B, & α Cen A/B. Rotational isochrones are drawn for ages of 226 Myr, 2.0 Gyr & 4.4 Gyr respectively, and the errors are indicated with dashed lines. Note that for all three wide binary systems, both components give substantially the same age. The dotted line corresponds to the age of the universe.

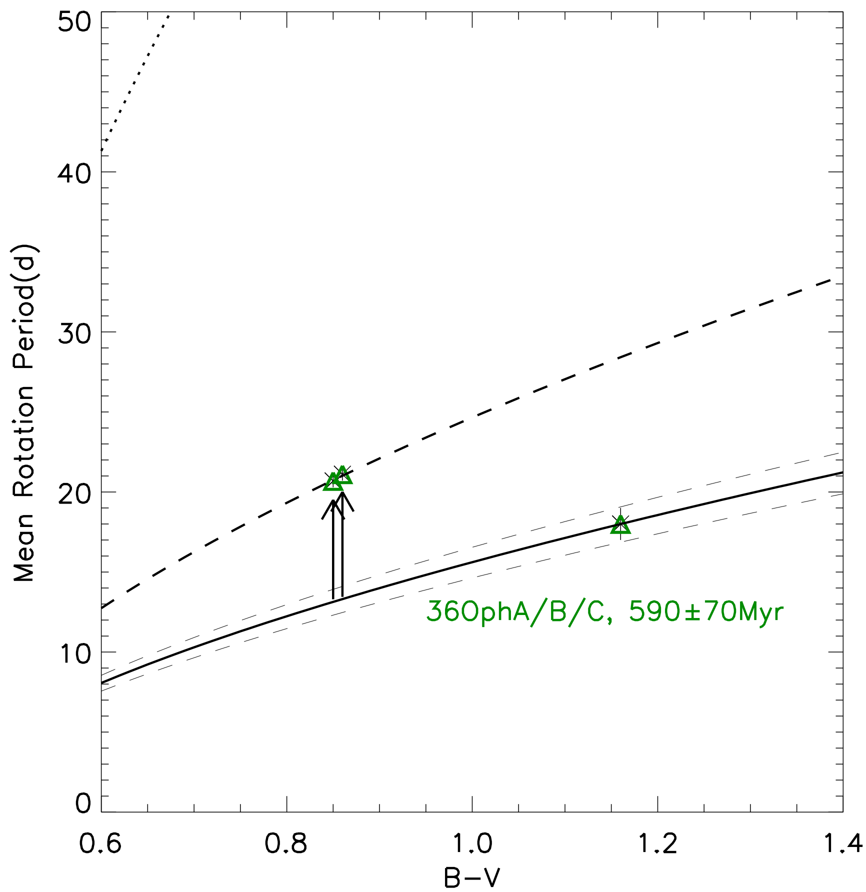

Fig. 17.— Color-period diagram for the 36 Oph ABC triple system. Isochrones are drawn for ages of 590 Myr (solid) and 1.43 Gyr (thick dashed). The distant companion, C, gives the 590 Myr age for the system. The error is indicated with thin dashed lines. The A and B components appear to have interacted and spun down to ∼20d against a nominally expected period of ∼13d. The dotted line corresponds to the age of the universe.

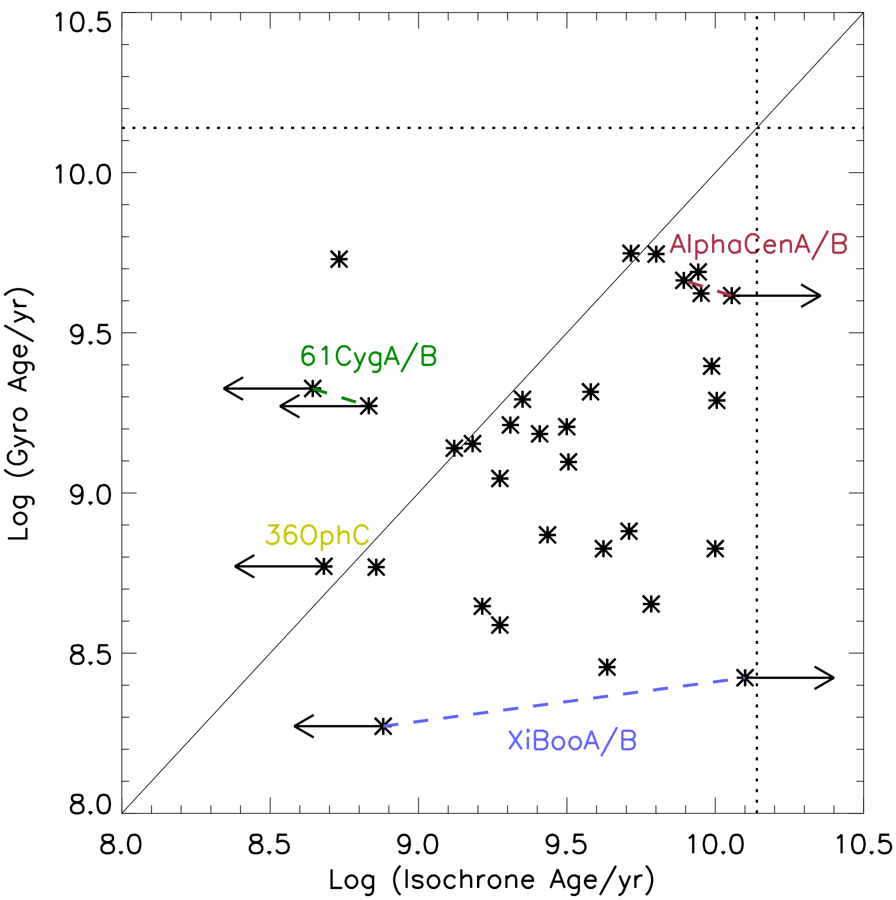

Fig. 18.— Comparison of gyro and isochrone ages for the 26 Takeda et al. (2007) stars with 'well-defined' ages in common with the gyrochronology sample presented in this paper. The solid line denotes equality and the dotted lines the age of the universe. There is no strong correlation between the two ages, except that the median isochrone age is a factor of 2.7 times higher than the median gyro age. Takeda et al. (2007) stars with upper- or lower limits (arrows) are not plotted, except for the wide binaries (the components are connected by dashed lines) discussed in the text. It would appear that the gyro ages supercede the isochrone ages for main sequence stars.